ELEMENTARY SCHOOL CHILDREN'S MATH UTILITY CONCEPTIONS: ASSOCIATIONS WITH PARENTS' CONCEPTIONS, MATH ACHIEVEMENT, AND HOME MATH ENGAGEMENT

By

Shari Renee Metzger

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, Baltimore County, in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2018

APPROVAL SHEET

Title of Dissertation: Elementary School Children's Math Utility Conceptions: Associations with Parents' Conceptions, Math Achievement, and Home Math Engagement

Name of Candidate: Shari Renee Metzger
Doctor of Philosophy, 2018

Dissertation and Abstract Approved: ______________________
Susan Sonnenschein, PhD
Professor
Applied Developmental Psychology

Date Approved: 8.27.18

NOTE: *The Approval Sheet with the original signature must accompany the thesis or dissertation. No terminal punctuation is to be used.

Curriculum Vitae

Shari Renee Metzger (Jurik)

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EDUCATION

Doctor of Philosophy, *UMBC*, Baltimore, MD, Applied Developmental Psychology, Anticipated Program Completion- December 2018

Bachelor of Arts, *Goucher College*, Baltimore, MD, Psychology, May 2007, 3.9 GPA, Honors in Psychology, Magna Cum Laude

TECHNICAL SKILLS

- Proficient with Word, Excel, PowerPoint, SPSS, Qualtrics, and NVIVO (qualitative data analytic software)
- Experience working with Excel Pivot tables, SAS, MPlus, and Stata

PROFESSIONAL EXPERIENCE

Research Analyst, **Research, Assessment, and Effectiveness (RAE),** Prince George's Community College, Largo, MD, May 2018-Present

- Supporting the assessment operations and federal and state reporting of the RAE office by retrieving data, conducting data analyses, designing and implementing research protocols, designing and administering surveys, conducting environmental scanning, and serving on campus-wide committees
- Preparing written analytical reports and presenting assessment results

Assessment Specialist Consultant, **Student Affairs,** UMBC, Baltimore, MD, January 2013-Present

- Supporting directors and staff in several departments by designing assessment plans, developing outcomes-based performance measures, and providing other assessment consultation as needed
- Leading staff development sessions and workshops about various assessment methods and tools

Graduate Assistant for Assessment, **Residential Life,** UMBC, Baltimore, MD, July 2011-October 2016

- Implemented nationally benchmarked resident and student staff assessments
- Developed assessments to measure the effectiveness of programs and services on department goals and divisional learning and service outcomes
- Used assessment results to effectively communicate a story, inspire action, and inform decisions within the department and with other campus stakeholders
- Developed, implemented, and evaluated annual assessment strategy for the department

Data Analysis Consultant, **Psychology Department,** UMBC, Baltimore, MD, Fall 2010-Summer 2011

- Performed and reported data analyses for course redesign evaluation for Developmental Psychology (PSYC 200)

Data Analysis Consultant, **Loyola College,** Baltimore, MD, 2007-2008

- Developed statistical analysis design and performed analyses for data collected by Dr. Cara Jacobson for her Psy.D. dissertation

RESEARCH & CLINICAL EXPERIENCE

Lab Manager, **UMBC,** Baltimore, MD, September 2009-May 2018

- Supervising and leading more than 25 undergraduate research assistants and three graduate students working on various projects
- Coordinating research and program evaluations with on- and off-campus research partners
- Designing and implementing quantitative and qualitative measures and analyzing data in several research studies
- Project manager for an evaluation of 21st century, an after-school program provided by the YMCA of Baltimore City for Moravia Elementary/Middle School

***Research Assistant*, UMBC,** Baltimore, MD, September 2008-May 2018

- Assisted in data collection and analysis for an evaluation of Core Knowledge, a Head Start curriculum implemented in two Head Start centers in Baltimore City
- Assisted in development of the evaluation report at the end of the third and fourth years of the evaluation

Intern, **Johns Hopkins Hospital East Baltimore Mental Health Partnership, School Based Program,** Waverly Elementary/Middle School, Baltimore, Maryland, Fall 2006

- Provided mental health therapy to students in the program, approximately 20 cases
- Observed and supported students in the classroom by managing behavior and helping them stay on task and understand the lessons
- Observed and participated in individual and group sessions with students
- Participated in Individual Educational Program (IEP) meetings and offered feedback from observations with the students to help determine the focus and progress of each IEP
- Supervisor, Ms. Margorie Gold, LCPC

Research Assistant, **Goucher College**, Baltimore, MD, Spring and Fall 2004

- Investigated strategies whereby students cope with threats to their psychological needs by creating, interpersonal disconnections, between themselves and their learning environments
- Analyzed the content of Dr. Patrick's research by identifying the subtle interpersonal motivations and strategies embedded in the text of the children's interviews
- Supervisor, Dr. Brian Patrick

TEACHING EXPERIENCE

***Instructor*, Introduction to Data Analysis Procedures for Industrial/Organizational Psychology (I/O Psychology Master's program),** UMBC-Shady Grove Campus, Rockville, MD, Fall 2017 and Fall 2018

***Teaching Assistant*, Statistical Analysis (Sociology Master's program),** UMBC, Baltimore, MD, Spring 2018

- Provided supplemental instruction and support to students when needed
- Graded assignments and exams
- Supervisor, Dr. Christine Mair

***Guest Lecturer*, Experimental Psychology: Design and Analysis I,** UMBC, Baltimore, MD, Spring 2012

- Topic- Null Hypothesis Testing

***Teaching Assistant*, Data Analytic Procedures I and II,** UMBC, Baltimore, MD, Fall 2010- Spring 2011

- Led laboratory portion of graduate statistics course, which involved preparing and teaching lessons and leading discussion
- Graded laboratory assignments and exams
- Supervisors, Dr. Steven Pitts, Dr. Laura Stapleton

***Teaching Assistant*, Experimental Psychology: Design and Analysis I,** UMBC, Baltimore, MD, Fall 2008- Spring 2010

- Led laboratory portion of undergraduate statistics course
- Graded laboratory assignments and exams
- Supervisors, Dr. Kimberly Warren, Dr. Lowell Groninger, Dr. Steven Pitts, Dr. Zoe Warwick

WORKSHOPS & GUEST PRESENTATIONS

***Invited Workshop*, Maryland Head Start Association Spring Conference,** McHenry, MD, May 2018

- Fostering Children's Math Development

***Invited Workshop*, Maryland Head Start Association Spring Conference,** Ocean City, MD, May 2017

- Empowering Parents to Foster Their Young Children's Math Development

***Invited Speaker*, Family Math Night,** Arlington Elementary/Middle School, Baltimore, MD, March 2017

***Professional Staff Development Presenter*, Division of Student Affairs,** UMBC, Baltimore, MD, 2011-2016

- Topics:
    - Using Direct and Indirect Measures for Program/Service Evaluation

- Evaluating Program Effectiveness
- Using Rubrics for Evaluating Staff Training
- Writing Good Survey Questions
- Using Qualitative Data to Make Decisions on Programs and Services
- Writing Good Learning Outcomes
- Tools for Assessment
- Data: You've got it, Now Use it…Effectively

PUBLICATIONS

Articles & Chapters:

- Sonnenschein, S., **Metzger, S. R.**, & Gay, B. (in press). Concerted cultivation among low-income Black and Latino families. In S. Sonnenschein, & B. E. Sawyer (Eds.), *Academic socialization of young Black and Latino children: Building on family strengths*. NY: Springer.
- Sonnenschein, S., Galindo, C., Simons, C. L., **Metzger, S. R.**, Thompson, J. A., & Chung, M. (2018). How do children learn mathematics? Chinese and Latino immigrant perspectives. In S.S. Chuang, & C. L. Costigan (Eds.), *Parental roles and relationships in immigrant families: An international approach.* New York, NY: Springer Science and Business Media.
- Dowling, R., **Metzger, S. R**., & Sonnenschein, S. (2018). Supporting young children's math development at home. (invited submission). *MSPA Protocol*
- Sonnenschein, S., **Metzger, S. R.,** Dowling, R., & Baker, L. (2017). The relative importance of English vs. Spanish language skills for low income Latino English language learners' early language and literacy development. *Early Child Development and Care*, *187*, 727-743. doi: 10.1080/03004430.2016.1219854 (to be reprinted in: R. Evans & O. N. Saracho (Eds.)). (in press, 2018) *Research in Young Children's Literacy and Language Development*. NY: Routledge, ISBN: 978-1-138-09109-2
- Sonnenschein, S., **Metzger, S. R.,** Dowling, R., Gay, B., & Simons, C. L. (2016). Extending an effective classroom-based math board game intervention to preschoolers' homes. *Journal of Applied Research on Children: Informing Policy for Children at Risk, 7*(2), 1-29. http://digitalcommons.library.tmc.edu/childrenatrisk/vol7/iss2/1
- Sonnenschein, S., **Metzger, S. R.,** & Thompson, J. A. (2016). Low-income parents' socialization of their preschoolers' early reading and math skills. *Research in Human Development, 13*, 207-224. doi: 10.1080/15427609.2016.1194707
- Sonnenschein, S., Stapleton, L.M., & **Metzger, S. R.** (2014). What parents know about how well their children are doing in school. *The Journal of Educational Research, 107,* 152-162.
- Sonnenschein, S., Thompson, J. A., **Metzger, S. R.**, & Baker, L. (2013). Relations between preschool teachers' language and gains in low income English language learners' and English speakers' vocabulary, early literacy and math skills. *NHSA Dialog: A Research- to-Practice Journal for the Early Childhood Field, 16*, 64-87.
- Sonnenschein, S., Thompson, J. A., **Metzger, S. R.**, & Baker, L. (2013). The importance of teachers' language and children's vocabulary to early academic skills. *NHSA Dialog: A Research- to-Practice Journal for the Early Childhood Field, 16,* 107-112.

- Sonnenschein, S., Galindo, C., **Metzger, S. R.**, Thompson, J.A., Huang, H. C., & Lewis, H. (2012). Parents' beliefs about children's math development and children's participation in math activities. *Child Development Research*, 13 pages. doi:10.1155/2012/851657
- Sonnenschein, S., & **Metzger, S. R.** (2012). Poverty, families, and schools. In J. A. Banks (Ed.), *Encyclopedia of Diversity in Education* (pp.1682-1685). Thousand Oaks, CA: Sage Publications, Inc.
- Mills, C. B., **Metzger, S. R.**, Foster, C., Valentine-Gresko, M., & Ricketts, S. (2009) Development of color-grapheme synesthesia and its effects on mathematical operations. *Perception, 38*, 591-605.

Program Evaluations:

- **Metzger, S. R.** (2017, July). *UMBC Recreation Services Assessment Report.* Sponsored by the Department of Athletics and Recreation and the Division of Student Affairs at UMBC.
- **Metzger, S. R.** (2016, May). *Final Report of Red Watch Band Program Year 1: Fall 2014-Spring 2015.* Sponsored by the NCAA CHOICES Alcohol Education Grant Program and UMBC.
- Sonnenschein, S., Baker, L., Thompson, J. A., & **Metzger, S. R.** (2011, January). *Evaluation of Core Knowledge Preschool Program at St. Vincent de Paul Head Start - Southeast: Year 4 Final Report*. Sponsored by the Abell Foundation.
- Sonnenschein, S., **Metzger, S. R.**, & Thompson, J. A. (2010, October). *Evaluation of 21st Century Community Learning Center Y Achievers Program at Moravia Park*. Sponsored by Maryland State Department of Education.
- Sonnenschein, S., Baker, L., Thompson, J. A., **Metzger, S. R.**, & Ramos, M. (2009, September). *Evaluation of Core Knowledge Preschool Program at St. Vincent de Paul Head Start - Southeast: Year 3*. Sponsored by the Abell Foundation.

Poster & Conference Presentations:

- Sonnenschein, S., Chen, Y., **Metzger, S. R.,** Simons, C. L., & Galindo, C. (2018, July). *Socialization of young children's math development: Chinese, Chinese-American, and U.S. White parents.* Poster presented at the International Society for the Study of Behavioural Development Biennial Meeting, Gold Coast, QLD, Australia.
- **Metzger, S. R.**, Gay, B., Dowling, R., & Sonnencshein, S. (2018, June). *Successes and challenges of extending an effective classroom-based math board game intervention to the home.* Paper presented in symposium (S. Sonnenschein, chair), *Promoting effective parent engagement in child's learning and social-emotional outcomes*, at the National Research Conference on Early Childhood, Arlington, VA.
- Sonnenschein, S., Dowling, R. & **Metzger, S. R.** (2018, May). *Fostering children's math development through play.* Invited workshop conducted at Maryland Head Start Association's Spring Conference, McHenry, MD.
- Sonnenschein, S., Simons, C. L., & **Metzger, S. R.** (2017, October). *Elementary school-aged children's knowledge of how they learn*. Paper presented in symposium (Sonnenschein, S. & Yamamoto, Y., co-chairs), *Perceptions of learning and school experiences from early childhood through adolescence*, at Society for the Study of Human Development, Providence, RI.

- Simons, C. L., Sonnenschein, S., Hill, R., Lee, A., & **Metzger, S. R.** (2017, August). *How children learn to use executive functioning strategies: Associations with memory task performance.* Poster presented at the American Psychological Association Convention, Washington, D.C.
- Sonnenschein, S., Dowling, R. & **Metzger, S. R.** (2017, May). *Empowering parents to foster their young children's math development.* Invited workshop conducted at Maryland Head Start Association's Spring Conference, Ocean City, MD.
- Simons, C. L., Sonnenschein, S., & **Metzger, S. R.** (2017, April). *Children's knowledge of their executive function strategies.* Poster presented at the Society for Research in Child Development Biennial Meeting, Austin, TX.
- **Metzger, S. R.,** Wohlstetter, G., Maier, T., & Jancuska, J. (2017, March). *UMBC recreation in review: Using data to keep students healthy, active, and engaged.* Poster presented at UMBC Student Affairs Assessment 360 Day, Baltimore, MD.
- **Metzger, S. R.** & Kottke, D. (2017, March). *Student affairs business service center: Integrations of systems to increase effectiveness and efficiency.* Poster presented at UMBC Student Affairs Assessment 360 Day, Baltimore, MD.
- Eby, D., Routzhan, C., & **Metzger, S. R.** (2017, March). *UMBC class of 2016 career outcomes.* Poster presented at UMBC Student Affairs Assessment 360 Day, Baltimore, MD.
- **Metzger, S. R.,** Dowling, R., Gay, B., Simons, C. L., & Sonnenschein, S. (2016, July). *Using parent focus groups to understand barriers to effective implementation of a home-based math intervention*. Poster presented at National Research Conference on Early Childhood, Washington, D.C.
- Sonnenschein, S., **Metzger, S. R.,** Thompson, J. A., & Gay, B. (2016, July). *Low income Black and Latino parents' concerted cultivation of preschoolers' reading and math skills.* Poster presented at the International Society for the Study of Behavioural Development Biennial Meeting, Vilnius, Lithuania.
- **Metzger, S. R.,** Sonnenschein, S., & Thompson, J. A. (2016, April). *Elementary school children's views about homework.* Poster presented at the American Educational Research Association Annual Meeting, Washington, D.C.
- **Metzger, S. R.,** & Caldwell, F. (2016, March). *Using data to implement and evaluate organizational change.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- **Metzger, S. R.,** & Wilson, J. A. (2016, March). *At-risk student learning: Direct and indirect measures for assessment.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- **Metzger, S. R.,** & Barnhart, D. (2016, March). *Exploring the experiences of transfer students of color at UMBC.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- **Metzger, S. R.,** & Wells, T. (2016, March). *Student Affairs Business Service Center: Using data about services to improve satisfaction.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- Mauriello, L., & **Metzger, S. R.** (2016, March). *Restorative practices at UMBC.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- Sonnenschein, S., **Metzger, S. R.,** & Thompson, J. A. (2015, October). *Parents' of low income preschoolers socialization of their children's early reading and math skills.*

Paper presented in symposium (Y. Yamamoto, Chair), *Parental academic socialization for childhood though college: Cultural and socioeconomic diversity,* Society for the Study of Human Development, Austin, TX.

- Singh, R., Chisolm, D. I., Simons, C. L., **Metzger, S. R.,** Sonnenschein, S. (2015, April). *The relationship between boy' and girls' self-awareness of learning strategies and their cognitive task performance.* Poster presented at the Undergraduate Research and Creative Achievement Day (URCAD), Baltimore, MD.
- **Metzger, S. R.,** Sonnenschein, S., Galindo, C., & Patel, H. (2015, March). *Children's beliefs about the utility of math and how these beliefs relate to their home math engagement.* Poster presented at the Society for Research in Child Development Biennial Meeting, Philadelphia, PA.
- **Metzger, S. R.,** Sonnenschein, S., Galindo, C., Thompson, J. A., & Simons, C. (2015, March). *Bringing a class-based math intervention to the home: The Importance of parents' beliefs*. Poster presented at the Society for Research in Child Development Biennial Meeting, Philadelphia, PA.
- Sonnenschein, S., **Metzger, S. R.,** Thompson, J. A., & Baker, L. (2015, March). *English and Spanish language predictors of low income Latino English language learners' early academic skills.* Poster presented at the Society for Research in Child Development Biennial Meeting, Philadelphia, PA.
- Galindo, C., Sonnenschein, S., Simons, C., Thompson, J. A., & **Metzger, S. R.,** (2015, March). *Chinese and Latino parents' views of how children learn math.* Poster presented at the Society for Research in Child Development Biennial Meeting, Philadelphia, PA.
- Simons, C., **Metzger, S. R.,** Higgins, C. Sonnenschein, S. (2015, March). *The nature of children's home engagement in math activities.* Poster presented at the Society for Research in Child Development Biennial Meeting, Philadelphia, PA.
- **Metzger, S. R.,** & Becks, R. (March, 2015). *Residential life work orders: Increasing efficiency and satisfaction.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- Wilson, J. A., & **Metzger, S. R.** (March, 2015). *Intent to return vs. license release data.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- Hague, J., & **Metzger, S. R.** (March, 2015). *Residential life student success: Developing intentional intervention efforts.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- Higgins, C., **Metzger, S. R.,** Simons, C.L., & Sonnenschein, S. (2014, November). *The nature of children's home-based math engagement*. Poster presented at the Annual Biomedical Research Conference for Minority Students (ABRCMS).
- Boone, K., & **Metzger, S. R.** (2014, July). *INTERACT: Developing students' authentic personal interactions.* Session presented at Association of College and University Housing Officers- International (ACUHO-I) Annual Conference and Exposition, Washington, D.C.
- Boone, K., & **Metzger, S. R.** (2014, March). *INTERACT: Developing students' authentic personal interactions.* Session presented at NASPA, Student Affairs Administrators in Higher Education, Baltimore, MD.
- **Metzger, S. R.** (2014, February). *Highlights and new factors on the fall 2013 Resident Assessment.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.

- **Metzger, S. R.,** & Wilson, J.A. (2014, February). *Why do students leave? Exit Interview Data: Trends and Triumphs.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- Mauriello, L., Hague, J., & **Metzger, S. R.** (2014, February). *Trends & interventions for at-risk residential students.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- **Metzger, S. R.,** & Treger, J. (2014, February). *Effectiveness of the Green Dot bystander intervention training program.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- **Metzger, S. R.** (2014, February). *EBI resident assessment—Institution specific questions.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- **Metzger, S. R.**, Sonnenschein, S., & Galindo, C. (2013, April). *Children's knowledge of mathematics and how math is used in daily activities.* Poster presented at the Society for Research in Child Development Biennial Meeting, Seattle, Washington.
- Sonnenschein, S., Galindo, C., Thompson, J. A., **Metzger, S. R.**, & Huang. H. C. (2013, April). *Chinese and White parents' beliefs about their children's math development.* Poster presented at the Society for Research in Child Development Biennial Meeting, Seattle, Washington.
- **Metzger, S. R.** (2013, February). *Important factors for resident student satisfaction.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- **Metzger, S. R.** (2013, February). *EBI resident assessment—Institution specific questions.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- Sonnenschein, S., **Metzger, S. R.**, Thompson, J. A., Lewis, H. (2012, June). *Similarities and differences in low-income African American and Hispanic parents' socialization of their children's academic development.* Poster will be presented at the annual Head Start Conference, Washington, DC.
- **Metzger, S. R.** (2012, February). *EBI resident assessment—Institution specific questions.* Poster presented at UMBC Student Affairs Data Day, Baltimore, MD.
- Sonnenschein, S., **Metzger, S. R.**, & Thompson, J. A. (2011, April). *Low income parents' socialization of their young children's reading and math development.* Poster presented at the Society for Research in Child Development Biennial Meeting, Montreal, Quebec, Canada.
- **Metzger, S. R.,** Sonnenschein, S., & Thompson, J. A. (2011, March). *The relation between parental beliefs about math, children's math activities and early math skills.* Poster presented at the Society for Research in Child Development Biennial Meeting, Montreal, Quebec, Canada.
- Sonnenschein, S., Thompson, J. A., **Metzger, S. R.**, & Baker, L. (2011, March). *Predictors of Head Start children's scores on authentic kindergarten school readiness measures.* Poster presented at the Society for Research in Child Development Biennial Meeting, Montreal, Quebec, Canada.
- Sonnenschein, S., Thompson, J. A., **Metzger, S. R.**, Ramos, M. F., & Baker, L. (2010, June). *Head Start children's math development: The role of classroom environment in facilitating English language learners' math skills.* Poster presented at the annual Head Start Conference, Washington, DC.

- Sonnenschein, S., Stapleton, L. M., & **Metzger, S. R.** (2009, April). *What do parents know about how well their children are doing in school?* Poster presented at the Society for Research in Child Development Biennial Meeting, Denver, Colorado.
- Sonnenschein, S., Ramos, M. F., Thompson, J. A., **Metzger, S. R.**, & Jackson, D. (2009, April). *Preschoolers' home-based reading and math activities.* Poster presented at the Society for Research in Child Development Biennial Meeting, Denver, Colorado.
- Mills, C. B., Ghirardelli, T. G., Zilioli, M., Bailey, L., & **Metzger, S.** (2008, March) *Additional evidence for faster processing of synesthetic colors in speeded classification of arithmetic equations.* Poster presented at annual conference of the Eastern Psychological Association, Boston, Massachusetts.
- Martinkowski, K, **Metzger, S. R.**, & Mills, C. B. (2007, March) *Neuropsychological evaluation of a color-digit synesthete.* Poster presented at annual conference of the Eastern Psychological Association, Philadelphia, Pennsylvania.
- Foster, C., Ricketts, S., **Metzger, S. R.**, & Mills, C. B. (2007, March) *A synesthete's solution: Wrong color + wrong color = slower times.* Poster presented at annual conference of the Eastern Psychological Association, Philadelphia, Pennsylvania.
- Valentine, M., Foster, C., **Metzger, S. R.**, & Mills, C. B. (2006, March) *Colored-digit synesthete and her experiences across languages.* Poster presented at annual conference of the Eastern Psychological Association, Baltimore, Maryland.

RELEVANT COURSEWORK & TRAINING

Data Analytic Procedures I
Data Analytic Procedures II
Causal Inferences in Program Evaluation
Structural Equation Modeling
Measurement of Behavior

Schooling and Development
Evaluating Interventions in Education
Applied Psychology and Public Policy
ACPA Assessment Institute- Student Affairs Assessment Training

PROFESSIONAL COMMITEES & MEMBERSHIPS

- ***American Educational Research Association (AERA),*** 2014- Present
- ***Golden Key International Honors Society,*** 2011- Present
- ***Society for Research in Child Development (SRCD)***, 2009- Present
- ***Phi Beta Kappa,*** **National Honors Society, Beta Chapter at Goucher College,** 2007- Present
- ***Psi Chi,*** **National Honor Society in Psychology,** 2005- Present
- ***UMBC Graduate Student Association,*** **Chair of Grants Selection Special Committee,** Baltimore, MD, Fall 2017-Spring 2018
- ***Board of Directors, Gateway Condominiums,*** **Treasurer,** Columbia, MD, May 2012- November 2016
- ***UMBC Student Affairs Assessment and Research Committee,*** Baltimore, MD, 2011- June 2016
- ***Student Affairs Administrators in Higher Education (NASPA),*** 2013- 2014
- ***UMBC Healthy Retrievers 2020 Assessment Committee,*** Baltimore, MD, 2013-2014

- ***UMBC Graduate Student Association Senate,* Senator for Applied Developmental Psychology,** Baltimore, MD, Fall 2009-Spring 2013
- ***Eastern Psychological Association,*** 2006-2008

AWARDS & SCHOLARSHIPS

***Dissertation Fellowship,* Graduate School, UMBC,** Fall 2017

- Awarded each semester to a select group of promising graduate students to provide them with financial support to devote necessary concentration and attention towards finishing the writing of their dissertations

***Excellence in Service Award,* Residential Life, UMBC,** Spring 2016

- Awarded annually to Residential Life staff members who demonstrate outstanding commitment to the Residential Life department and support of student success and development

***The Ruth C. Wylie Prize*, Department of Psychology, Goucher College,** Spring 2007

- Awarded annually to a senior psychology major who best exemplifies a promising psychology student, both academically and professionally

***Global Citizen Scholarship*, Goucher College,** Fall 2003-Spring 2007

- Awarded to first-year applicants and renewed for students who demonstrate potential for outstanding academic achievement at Goucher and for contributing to a vibrant campus community

***Scholarship,* Federal Employee Education and Assistance Fund,** Fall 2006-Spring 2007

- Merit-based scholarship awarded to dependents of Federal Employees for outstanding academic records, character references, and essay

***Scholarship,* Delegate Carol S. Petzold,** Fall 2003- Spring 2004, Fall 2005-Spring 2007

- Merit-based scholarship awarded to residents of Maryland in District 19 with excellent academic standing

ABSTRACT

Title of Document: ELEMENTARY SCHOOL CHILDREN'S MATH UTILITY CONEPTIONS: ASSOCIATIONS WITH PARENTS' CONCEPTIONS, MATH ACHIEVEMENT, AND HOME MATH ENGAGEMENT.

Shari Renee Metzger, Ph.D., 2018

Directed By: Professor Susan Sonnenschein, Department of Psychology

This dissertation was made up of three studies, with the overall goal of examining first through fourth grade children's math utility conceptions—knowledge and beliefs about the usefulness of math—and how those conceptions relate to their parents' conceptions and children's math achievement. All three studies used the *Math Utility Conceptions* conceptual model, an expansion of multiple theoretical models, to investigate children's math utility conceptions.

The first paper examined children's math utility conceptions and grade-level differences in math utility conceptions and home math engagement. Most children viewed math as heavily focused on low-level math operations and as learned and used primarily in school. Older children had more awareness of math in daily activities, but had a more school-based view than younger children. The second paper primarily investigated the associations between parents' and children's math utility conceptions and children's home math engagement. Parents' math utility conceptions positively predicted children's math utility conceptions; this relation was moderated by the frequency with which children engaged in math activities at home and how often children see their parents using math. Results suggest that children develop their knowledge and beliefs

about math utility from their parents as well as through engagement in math-related activities. The third paper explored the relation between children's math utility conceptions and their math achievement. Overall math utility conceptions predicted math reasoning skills. Children's productive disposition significantly predicted their math achievement. The associations between math applicability, math utility, and math achievement were different for older and younger children. Results suggest that the relation between children's math utility conceptions is complex, but the extent to which elementary-age children view math as useful and worthwhile is associated with children's math achievement.

Overall, these results may guide math curriculum development for elementary-aged children to more intentionally use real-world applications to teach math concepts and, in doing so, improve children's understanding of the importance of math in their daily lives. By increasing young children's knowledge of applications of math outside the school context and beliefs about the usefulness of math, parents and educators could help to increase children's math proficiency.

Acknowledgements

I would like to offer great thanks to everyone who helped me throughout my journey at the University of Maryland, Baltimore County. With the support of faculty, staff, graduate students, undergraduate students, research partners, and my family, I was able to gain a wealth of knowledge and experiences that will guide my future career. I would like to specifically thank my advisor and mentor, Dr. Susan Sonnenschein for her guidance through my graduate career. She presented me with an idea to study what children think about math and helped me to develop an interest and a passion for this research area that will continue into my career. The variety of opportunities she provided in her research lab allowed me to cultivate skills that I use in my job every day. Also, as the Graduate Program Director, I always knew that she was my advocate as I navigated new areas of my graduate experience.

I would also like to thank my committee members, Drs. Linda Baker, Shuyan Sun, Claudia Galindo, and Geetha Ramani. I greatly appreciate their time, flexibility, and excellent feedback that has greatly improved the quality of my dissertation work. Their varying perspectives and ideas were invaluable to strengthening my research and preparing my work for publication. I also want to thank the families that participated in my research, the graduate students in my lab who supported me through data collection and analysis, and the undergraduate research assistants who contributed to the completion of these studies.

Finally, I am so grateful to my family and friends who supported me throughout my graduate career. The unwavering support from my parents, Stuart and Ruth Metzger, sister, Heather Metzger, and husband, Andrew Jurik, helped me get through the struggles and made the triumphs even more exciting. I credit a large part of where I am today to their love and encouragement.

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Chapter 1: General Introduction

The achievement gap in math between the United States and many Asian and European nations is well-documented (e.g., National Mathematics Advisory Panel, 2008; U.S. Department of Education, 2011, 2013, 2014, 2015a, 2015b). For example, in 2015, U.S. fourth and eighth graders were ranked 10th in international benchmarks for math. With the United States behind in the international fields of mathematics, we need to find ways to raise national standards and performance of the nearly 50 million children currently enrolled in schools in the U.S. In order to increase the number of children in the United States who are able to demonstrate proficiency in mathematics, it is important to determine factors that contribute to mathematical learning and expertise.

One factor that is associated with mathematical learning is children's math conceptions (Muis, 2004; National Research Council, 2001), which include children's knowledge, attitudes, and beliefs about math. Research shows that children's perceived value of math, an aspect of math conceptions, is positively related to math achievement (Aunola, Leskinen, & Nurmi, 2006; Eccles, Wigfield, Harold, & Blumenfeld, 1993; Marsh, Trautwein, Lüdtke, Köller, & Baumert, 2005; Mason, 2003; Mazzocco, Hanich, & Noeder, 2012; Pajares & Miller, 1994; Schoenfeld, 1989). Researchers have examined a variety of aspects of children's conceptions about math and their engagement in math activities. One such dimension, math utility, refers to the usefulness of math in school and daily life. Eccles and colleagues (e.g., Eccles & Wigfield, 2002; Eccles et al., 1993; Muenks, Wigfield, & Eccles, 2018; Musu-Gillette, Wigfield, Harring, & Eccles, 2015) included utility-value within their expectancy-value theory of motivation model, because beliefs about the usefulness of math are an important part of the value children place on math tasks.

Math utility-value refers to how much children believe that math is useful and important for their lives.

Little research has examined in-depth elementary-age children's conceptions of math utility, specifically looking at both their knowledge about math utility and beliefs about the usefulness of math. The National Research Council (2001) focused on the importance of math utility within their five strands of math proficiency model. In this model, productive disposition, one of the five strands, refers to children's beliefs that math is useful and worthwhile. Because early math skills predict later math achievement (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Duncan, et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; LeFevre, Polyzoi, Skwarchuck, Fast, & Sowinski, 2010), understanding elementary-age children's conceptualizations of math utility is important for improving children's future math achievement. The three studies in this dissertation investigated children's math utility beliefs, how they are acquired, and how these beliefs relate to math achievement.

These three studies expanded on the way that math utility conceptions are measured. In the conceptual model, children's math utility conceptions consist of children's *knowledge about math utility*, including math concepts and applicability of math, and *beliefs about math utility*, including utility value and productive disposition (for a visual representation of the model, see Figure 1). *Knowledge about math utility* refers to children's understanding of what math is and how it can be used by themselves and others within and outside of the school context. *Beliefs about math utility* refers to children's affective beliefs regarding the usefulness and worthwhile nature of math. This model expanded on other conceptual models (e.g., Eccles et al., 1993) by including both knowledge *and* belief components of conceptions. This is important because research shows that children's ability to assess their knowledge of math is related to their math learning and achievement (Dunlosky & Rawson, 2012; Vo, Li, Kornell, Pouget, & Cantlon, 2014).

The Development of Math Conceptions

An aspect of math utility conceptions that has not yet been studied is how parents' conceptions about math utility are associated with their children's conceptions. Young children's home numeracy environments are associated with their early math skills (Downer & Pianta, 2006; LeFevre et al., 2009; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Manolitsis, Georgiou, & Tziraki, 2013). Young children acquire mathematics knowledge even before they start school (Ginsburg, Duch, Ertle, & Noble, 2012; Ginsburg, Lee, & Boyd, 2008; Sarama & Clements, 2007; Siegler & Mu, 2008). Engaging in everyday math activities, including playing board games, positively predicts children's math skills (Blevins-Knabe & Musun-Miller, 1996; Clements & Sarama, 2006; Ramani & Siegler, 2008). If parents believe it is their role to engage their children in math activities at home, they may encourage their children to engage in more activities (Sonnenschein et al., 2012; Sonnenschein, Metzger, & Thompson, 2016). However, Metzger, Sonnenschein, and Galindo (2018, Study 1) found that frequency of engagement in math-related activities at home did not relate to children's awareness of how math features in daily activities. Thus, other aspects of children's numeracy environment, such as parents' conceptions and aspects of the home numeracy environment, may impact the development of children's math utility conceptions.

Math Conceptions and Achievement

Another important research consideration is to examine how math utility conceptions are associated with children's math achievement. Researchers have studied extensively the relation between math motivations and math achievement with elementary-age children (see Wigfield, et al., 2015; Wigfield, Eccles, Roeser, & Schiefele, 2008; Wigfield, Eccles, Schiefele, Roeser, & Davis-Kean, 2006 for reviews). Children's motivations for math are related to their math achievement (e.g., Aunola et al., 2006; Corbière, Fraccaroli, Mbekou, & Perron, 2006; De Corte & Verschaffel, 2006; Eccles et al., 1993; Gal & Ginsburg, 1994; Marsh et al., 2005; Mason, 2003; Muis, 2004; Pajares & Miller, 1994; Shores & Shannon, 2007; Wigfield et al., 2006). Less is known about math utility conceptions, how children view math as useful outside of the school context, and how those conceptions relate to the development of children's early math skills (De Corte, Op't Eynde, & Verschaffel, 2002; Eccles & Wigfield, 2002; Eccles et al., 1993; Schiefele & Csikszentmihalyi, 1995).

Eccles and colleagues (e.g., Eccles & Wigfield, 2002; Eccles et al., 1989; Eccles et al., 1993; Wigfield et al., 1997) developed an expectancy-value theory which includes subjective task value, comprised of both interest and perceived usefulness. Research has shown that children who believe that mathematics is interesting, fun, useful, and important are more likely to engage in mathematical activities and have higher math achievement (De Corte et al., 2002; Eccles & Wigfield, 2002; Eccles et al., 1993; Fisher, 2004; Gottfried, Fleming, & Gottfried, 2001; Gottfried, Marcoulides, Gottfried, Oliver, & Guerin, 2007; Mason, 2003; Mazzocco et al., 2012; Murayama, Pekrun, Lichtenfeld, & vom Hofe, 2013; Schiefele & Csikszentmihalyi, 1995; Schiefele, Krapp, & Winteler, 1992). The National Research Council (2001) posited that developing a productive disposition towards math is an integral part of developing math proficiency. However, little research has examined the relations between productive disposition and other math competencies in their model. The limited research on this topic shows that children do not often define math as being used outside the classroom (Mazzocco et al., 2012; Perlmutter, Bloom, Rose, & Rogers, 1997), but their beliefs about the usefulness of math positively predict later math achievement (Mazzocco et al., 2012). It is important to study these relations further in order to better understand how math utility conceptions may impact children's math achievement.

The Present Studies

All three studies used the *Math Utility Conceptions* conceptual model to investigate children's math utility conceptions. These studies built on previous research by expanding on studies that included only a few math utility beliefs items (Eccles et al., 1993; Mazzocco et al., 2012; Perlmutter et al., 1997). Also, children and parents were asked to describe what they believe math is as well as how it is used in various contexts.

Study 1 examined children's math utility conceptions using the above model. It also examined children's home-based math engagement, how engagement relates to math utility conceptions, and grade-level differences in math utility conceptions and home engagement. Most children viewed math as heavily focused on low-level math operations and as learned and used primarily in school. Older children had a more complex conceptualization of math utility, characterized by more awareness of math as part of daily living and a view of math as more school-based than their younger counterparts. Results suggest that children's awareness of math in daily activities may be associated with children's conceptions about math.

Study 2 investigated studied children's math utility conceptions in a new sample of rising first through fourth graders and explored the associations between parents' and children's math utility conceptions and children's home math engagement. Consistent with Study 1, children primarily viewed math as school-based. Parents' math utility conceptions positively predicted children's math utility conceptions; and this relation was moderated by the frequency with which children engaged in math activities at home and how often children see their parents using math.

Home math engagement was not a direct predictor of children's math utility conceptions. Results suggest that children develop their knowledge and beliefs about math utility from their parents as well as through engagement in math-related activities.

Study 3 explored the relation between children's math utility conceptions and their math achievement using the same sample from Study 2. Consistent with the National Research Council's (2001) model, children's productive disposition significantly predicted their math achievement. Overall math utility conceptions did not predict math achievement. However, the associations between math applicability, math utility, and math achievement were different directions for different age groups. Results suggest that the relation between children's math utility conceptions is complex, but the extent to which elementary-age children view math as useful and worthwhile is associated with children's math achievement.

References

Aunola, K., Lesiken, E., Lerkkanen, M. K., & Nurmi, J. E. (2004). Developmental dynamics of math performance from preschool to Grade 2. *Journal of Educational Psychology, 96*, 699-713. doi:10.1037/0022-0663.96.4.699

Aunola, K., Leskinen, E., & Nurmi, J.-E. (2006). Developmental dynamics between mathematical performance, task motivation, and teachers' goals during transition to primary school. *British Journal of Educational Psychology, 76*, 21-40. doi:10.1348/000709905X51608

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting, 5,* 35-45. doi:10.1002/(SICI)1099-0917(199603)5:1<35::AID-EDP113>3.0.CO;2-0

Clements, D. H., & Sarama, J. (2006). Your child's mathematical mind. *Scholastic Parent & Child*, *14*, 30-39.

Corbière, M., Fraccaroli, F., Mbekou, V., & Perron, J. (2006). Academic self-concept and academic interest measurement: A multi-sample European study. *European Journal of Psychology of Education*, *21*, 3-15.

De Corte, E., Op't Eynde, P., & Verschaffel, L. (2002). "Knowing what to believe": The relevance of students' mathematical beliefs for mathematics education. In B. K. Hofer & P. R. Pintrich (Eds.), *Personal epistemology: The psychology of beliefs about knowledge and knowing* (pp. 297-320). Mahwah, NJ: Lawrence Erlbaum Associates.

De Corte, E., & Verschaffel, L. (2006). Mathematical thinking and learning. In W. Damon & R. M. Lerner (Series Eds.) & K. A. Renninger & I. E. Sigel (Vol. Eds.), *Handbook of Child Psychology: Vol. 4 Child psychology in practice* (6^{th} ed., pp. 103-152). New York: Wiley.

Downer, J. T., & Pianta, R. C. (2006). Academic and cognitive functioning in first grade: Associations with earlier home and child care predictors and with concurrent home and classroom experiences. *Social Psychology Review, 35*, 11-30.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., … Japel, C. (2007). School readiness and later achievement. *Developmental Psychology, 43*, 1428-1446. doi:10.1037/0012-1649.43.6.1428.supp

Dunlosky, J., & Rawson, K. A. (2012). Overconfidence produces underachievement: Inaccurate self evaluations undermine students' learning and retention. *Learning and Instruction*, *22*, 271-280. doi:10.1016/j.learninstruc.2011.08.003

Eccles, J. S., & Wigfield, A. (2002). Motivational beliefs, values, and goals. *Annual Review of Psychology, 53*, 109-132. doi:10.1146/annurev.psych.53.100901.135153

Eccles, J. S., Wigfield, A., Flanagan, C. A., Miller, C., Reuman, D. A., & Yee, D. (1989). Self-concepts, domain values, and self-esteem: Relations and changes at early adolescence. *Journal of Personality, 57*, 283-310. doi:10.1111/j.1467-6494.1989.tb00484.x

Eccles, J. S., Wigfield, A., Harold, R., & Blumenfeld, P. (1993). Age and gender differences in children's self-and task perceptions during elementary school. *Child Development, 64*, 830-847. doi:10.1111/j.1467-8624.1993.tb02946.x

Fisher, P. H. (2004). *Early math interest and the development of math skills: An understudied relationship.* Unpublished doctoral dissertation, University of Massachusetts Amherst,

Amherst, MA. (ProQuest Information and Learning Company) Retrieved September 1, 2009 from ProQuest database.

Gal, I., & Ginsburg, L. (1994). The role of beliefs and attitudes in learning statistics: Towards an assessment framework. *Journal of Statistics Education*, *2*, 1-15. doi:10.1080/10691898.1994.11910471

Ginsburg, H. P., Duch, H., Ertle, B., & Noble, K. G. (2012). How can parents help their children learn math?. B. H. Wasik (Ed.), *Handbook of Family Literacy* (2nd ed., pp. 51-65). New York: Routledge.

Ginsburg, H. P., Lee, J. S., Boyd, J. S. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report, 22,* 3-22.

Gottfried, A. E., Fleming, J. S., & Gottfried, A. W. (2001). Continuity of academic intrinsic motivation from childhood through late adolescence: A longitudinal study. *Journal of Educational Psychology*, *93*, 3-13. doi:10.1037//0022-0663.93.1.3

Gottfried, A. E., Marcoulides, G. A., Gottfried, A. W., Oliver, P. H., & Guerin, D. W. (2007). Multivariate latent change modeling of developmental decline in academic intrinsic math motivation and achievement: Childhood through adolescence. *International Journal of Behavioral Development*, *31*, 317-327. doi:10.1177/0165025407077752

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, *45*, 850-867. doi:10.1037/a0014939

LeFevre, J.-A., Polyzoi, E., Skwarchuk, S.-L., Fast, L., & Sowinski, C. (2010). Do home numeracy and literacy practices of Greek and Canadian parents predict the numeracy skills of kindergarten children? *International Journal of Early Years Education, 18*, 55-70. doi:10.1080/09669761003693926

LeFevre, J.-A., Skwarchuk, S.-L., Smith-Chant, B.L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children's math performance in the early school years. *Canadian Journal of Behavioral Science, 41*, 55-66. doi:10.1037/a0014532

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology*, *46*, 1309-1319. doi:10.1037/a0019671

Manolitsis, G., Georgiou, G. K., & Tziraki, N. (2013). Examining the effects of home literacy and numeracy environment on early reading and math acquisition. *Early Childhood Research Quarterly*, *2013*, 692-703. doi:10.1016/j.ecresq.2013.05.004

Marsh, H. W., Trautwein, U., Lüdtke, O., Köller, O., & Baumert, J. (2005). Academic self-concept, interest, grades, and standardized test scores: Reciprocal effects models of causal ordering. *Child Development, 76*, 397-416. doi:10.1111/j.1467-8624.2005.00853.x

Mason, L. (2003). High school students' beliefs about maths, mathematical problem solving, and their achievement in maths: A cross-sectional study. *Educational Psychology, 23*, 73-85. doi:10.1080/01443410220000022942

Mazzocco, M. M., Hanich, L. B., & Noeder, M. M. (2012). Primary school age students' spontaneous comments about math reveal emerging dispositions linked to later mathematics achievement. *Child Development Research*, *2012,* doi:10.1155/2012/170310.

Metzger, S. R., Sonnenschein, S., & Galindo, C. (2018). *Elementary-age children's conceptions about math utility and their home-based math engagement.* Manuscript submitted for publication.

Muenks, K., Wigfield, A., & Eccles, J. S. (2018). I can do this! The development and calibration of children's expectations for success and competence beliefs. *Developmental Review*. doi:10.1016/j.dr.2018.04.001

Muis, K. R. (2004). Personal epistemology and mathematics: A critical review and synthesis of research. *Review of Educational Research, 74*, 317-377. doi:10.3012/00346543074003317

Murayama, K., Pekrun, R., Lichtenfeld, S., & vom Hofe, R. (2013). Predicting long-term growth in students' mathematics achievement: The unique contributions of motivation and cognitive strategies. *Child Development*, *84*, 1475-1490. doi:10.1111/cdev.12036

Musu-Gillette, L. E., Wigfield, A., Harring, J. R., & Eccles, J. S. (2015). Trajectories of change in students' self-concepts of ability and values in math and college major choice. *Educational Research and Evaluation*, *21*, 343-370. doi:10.1080/13803611.2015.1057161

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences of Education. Washington, DC: National Academy Press.

National Mathematics Advisory Panel (2008). Foundations for success: The final report of the National Mathematics Advisory Panel.

Pajares, F., & Miller, M. D. (1994). Role of self-efficacy and self-concept beliefs in mathematical problem solving: A path analysis. *Journal of Educational Psychology, 86*, 193-203.

Perlmutter, J., Bloom, L., Rose, T., & Rogers, A. (1997). Who uses math? Primary children's perceptions of the uses of mathematics. *Journal of Research in Childhood Education, 12*, 58-70.

Ramani, G. B & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development, 79*, 375-394. doi:10.1111/j.1467-8624.2007.01131

Sarama, J. & Clements, D. H. (2007). How children problem solve. *Early Childhood Today, 21*, 16-19.

Schiefele, U., & Csikszentmihalyi, M. (1995). Motivation and ability as factors in mathematics experience and achievement. *Journal for Research in Mathematics Education, 26,* 163-181. doi:10.2307/749208

Schiefele, U., Krapp, A., & Winteler, A. (1992). Interest as a predictor of academic achievement. In K.A. Renninger, S. Hidi, & A. Krapp (Eds.), *The role of interest in learning and development* (pp.183-212). Hillsdale, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. H. (1989). Explorations of students' mathematical beliefs and behavior. *Journal in Research in Mathematics Education, 20,* 338-355. doi:10.2307/749440

Shores, M. L., & Shannon, D. M. (2007). The effects of self-regulation, motivation, anxiety, and attributions on mathematics achievement for fifth and sixth grade students. *School Science and Mathematics, 107*, 225-236. doi:10.1111/j.1949-8594.2007.tb18284.x

Siegler, R. S., & Mu, Y. (2008). Chinese children excel on novel mathematics problems even before elementary school. *Psychological Science, 19*, 759-763. doi:10.1111/j.1467-9280.2008.02153.x

Sonnenschein, S., Galindo, C., Metzger, S. R., Thompson, J. A., Huang, H. C., & Lewis, H. (2012). Parents' beliefs about children's math development and children's participation in math activities. *Child Development Research, 2012*, doi:10.1155/2012/851657

Sonnenschein, S., Metzger, S. R., & Thompson, J., A. (2016). Low-income parents' socialization of their preschoolers' early reading and math skills. *Research in Human Development, 13*, 207-224. doi: 10.1080/15427609.2016.1194707

U.S. Department of Education. (2011). *Trends in International Math and Science Study*. National Center for Educational Statistics.

U.S. Department of Education. (2013). *National Assessment of Education Progress.* National Center for Educational Statistics.

U.S. Department of Education. (2014). *2013 Grade 12 Reading and Mathematics Results: National Assessment of Education Progress.* National Center for Educational Statistics.

U.S. Department of Education. (2015a). *National Assessment of Education Progress.* National Center for Educational Statistics.

U.S. Department of Education. (2015b). *Trends in International Math and Science Study*. National Center for Educational Statistics.

Vo, V. A., Li, R., Kornell, N., Pouget, A., & Cantlon, J. F. (2014). Young children bet on their numerical skills metacognition in the numerical domain. *Psychological Science*, *25*, 1712-1721. doi:10.1177/0956797614538458

Wigfield, A., Eccles, J. S., Fredricks, J. A., Simpkins, S., Roeser, R. W., & Schiefele, U. (2015). Development of achievement motivation and engagement. In R. M. Lerner (Ed.) & M. E. Lamb (Vol. Ed.), *Handbook of child psychology and developmental science: Vol. 3 socioemotional processes* (7th ed., pp. 1-44). Hoboken, NJ: John Wiley & Sons, Inc.

Wigfield, A., Eccles, J. S., Roeser, R. W., & Schiefele, U. (2008). Development of achievement motivation. In W. Damon, R. M. Lerner, D. Kuhn, R. S. Siegler, & N. Eisenberg (Eds.), *Child and adolescent development: An advanced course* (pp. 406-434). Hoboken, NJ: John Wiley & Sons, Inc.

Wigfield, A., Eccles, J. S., Schiefele, U., Roeser, R. W., & Davis-Kean, P. (2006). Development of achievement motivation. In W. Damon & R. M. Lerner (Series Eds.) & D. Kuhn & R. Siegler (Vol. Eds.), *Handbook of child psychology: Vol. 3 social, emotional, and personality development* (6th ed., pp. 933-1002). New York: Wiley.

Wigfield, A., Eccles, J. S., Yoon, K. S., Harold, R. D, Arbreton, A. J. A., Freedman-Doan, C. & Blumenfeld, P. C. (1997). Change in children's competence beliefs and subjective task values across the elementary school years: A 3-year study. *Journal of Educational Psychology, 89*, 451-469. doi:10.1037/0022-0663.89.3.451

Chapter 2

Elementary-Age Children's Conceptions about Math Utility and their Home-Based Math Engagement

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Abstract

Integrating multiple theoretical frameworks, this study examined rising first through fourth grade children's math utility conceptions—their knowledge and beliefs about the usefulness of math, home-based math engagement, and grade-level differences in math utility conceptions and home engagement. Most children viewed math as heavily focused on low-level math operations and as learned and used primarily in school. Older children showed more awareness of math as part of daily living, but still viewed math as mostly school-based—more so than their younger counterparts. Results suggest that awareness of math in daily life may be associated with children's math utility value (the perceived usefulness of math). Although children engaged in activities at home with the potential to foster math development, the frequency of engagement was not related to their awareness of math in daily activities. Thus, there may be untapped opportunities for young children to connect the math they learn in school to their daily life.

Many children in the United States earn low scores on standardized math assessments (National Mathematics Advisory Panel, 2008). For example, only 40% of U.S. fourth graders, 34% of eighth graders, and 25% of twelfth graders in 2017 scored in the proficient or advanced range on the 2017 National Assessment of Educational Progress (NAEP) mathematics assessment (U.S. Department of Education, 2018). Given the importance of math for subsequent academic and vocational success (Clark, 1988; National Mathematics Advisory Panel, 2008), it is critical to understand factors associated with children's math learning, especially those outside the school context.

Research on children's math learning outside of school generally focuses either on the specific math activities they engage in (e.g., Blevins-Knabe & Musin-Miller, 1996; LeFevre et al., 2009; Niklas & Schneider, 2014) or their math self-concepts (e.g., Muenks, Wigfield, & Eccles, 2018; Musu-Gillette, Wigfield, Harring, & Eccles, 2015; Schoenfeld, 1992). Far less research has focused on children's understanding of what math is, how they use math in daily activities, and how math knowledge is acquired. This study focuses on children's conceptions about math and their engagement in math activities at home. More specifically, it addresses children's math utility conceptions, that is, how much children believe that math is useful and important in their lives and how they believe they acquire that knowledge. It also examines grade-level differences (rising first through fourth grades) in math utility conceptions and home engagement.

Theoretical Approach to Math Utility Conceptions

We conceptualize math utility conceptions as a multi-dimensional construct formed by two dimensions: children's *knowledge about math* and *beliefs about math utility*. *Knowledge about math* refers to the extent of children's knowledge of the aspects of mathematics (math concepts) and the ways in which math can be used by themselves and others across different contexts (applicability of math). *Beliefs about math utility* refers to children's beliefs about the usefulness of math (utility value and productive disposition; see Figure 1 for a visual representation). An important distinction between these two dimensions is that *knowledge about math* assesses how much children know about the breadth of math and the potential uses of math in various daily activities and *beliefs about math utility* assesses value that children place on math for themselves and others.

Knowledge about math. Children's ability to assess their own knowledge of math is related to their math learning and achievement (Dunlosky & Rawson, 2012; Vo, Li, Kornell, Pouget, & Cantlon, 2014). A growing body of research has examined the development of children's mathematical knowledge both in school and at home (Browning et al., 2016; Krawec, Huang, Montague, Kressler, & de Alba, 2013; Rosenzweig, Krawec, & Montague, 2011; Van Oers, 2010).

***Math concepts*.** Children need to understand what math is before they can develop conceptions about its usefulness. Consistent with the most recent version of the National Council of Teachers of Mathematics math standards (NCTM, 2000), the conceptual model in this study views math knowledge or concepts as consisting of content and processes. Content includes number and operations, algebra, geometry, measurement, and data analysis and probability. Processes include problem solving, reasoning and proof, communication, connections, and representations. Although these standards were published nearly two decades ago, they remain an integral part of current mathematics curricula. The NCTM (2000) standards, along with the National Research Council's (2001) model for developing mathematical proficiency, were the foundation for the creation of the Common Core State Standards for Math (CCSSI, 2010;

Kendall, 2011). In addition, children's content and process knowledge build on and influence the development of one another; both are considered critical for the development of math proficiency (Rittle-Johnson, 2017).

To the best of our knowledge, only two studies, Perlmutter, Bloom, Rose, and Rogers (1997) and Mazzocco, Hanich, and Noeder (2012), have investigated kindergarten through third grade children's knowledge of what math is. Perlmutter et al. (1997) found that these children's definitions of math consisted primarily of number and operations. Mazzocco et al. (2012) coded responses of second and third grade children's descriptions of what math is using a 5-point scale that ranged from irrelevant responses to responses where children defined math as a useful tool. Children's definitions included mainly basic math principles or mechanics (i.e., numbers and operations). Unfortunately, coding of children's definitions of math in both studies did not include math concepts and processes, both of which are important for the development of math knowledge (Rittle-Johnson, 2017). The present study provides a broader scope of children's definitions of math by aligning coding for these responses to the NCTM's (2000) math content and process standards.

Applicability of math. In spite of the importance of children's conceptions of math (De Corte & Verschaffel, 2006; Muis, 2004) and recent efforts to better understand the role of math utility in children's math development (Mazzocco et al., 2012; Rozek, Hyde, Svoboda, Hulleman, & Harackiewicz, 2015), we still know fairly little about children's understanding of how math can be applied in different contexts. Much of the current research about knowledge of the applicability of math in daily life has been conducted with high school and college students and shows that learning math through "real-world" applications is positively associated with using math to solve real-world problems (Barab, Squire, & Dueber, 2000; Herrington, Reeves, &

Oliver, 2013; National Mathematics Advisory Panel, 2008). For example, Herrington et al. (2013) and Barab et al. (2000) found that when college students were taught in authentic learning environments (specific real-world contexts), they were better able to integrate and apply this knowledge in their daily lives. Hulleman and colleagues (Hulleman & Harackiewicz, 2009; Hulleman, Kosovich, Barron, & Daniel, 2017) found that utility value interventions designed to increase the connections that high school and college students made between course material and their lives increased how much students valued the course and their performance, especially for the lowest performing students.

Little research has examined connections between school math and math in daily activities with younger children. Perlmutter et al. (1997) asked children about the usefulness of math for cooking and going to the grocery store. Although children were aware of some uses for math in those activities, their awareness was very limited. Children in kindergarten who were taught using the Realistic Mathematics Education curriculum, which presents math problems using daily activities like visiting the grocery store or a museum, showed significantly greater growth in early math skills than children taught with the standard curriculum (Papadakis, Kalogiannakis, & Zaranis, 2017).

Beliefs about math utility. Children's beliefs about math are associated with their early math skills (see De Corte & Verschaffel, 2006; Marsh, Trautwein, Lüdtke, Köller, & Baumert, 2005; Wigfield et al., 2015; Wigfield, Eccles, Roeser, & Schiefele, 2008; Wigfield, Eccles, Schiefele, Roeser, & Davis-Kean, 2006), which, in turn, are related to later math achievement (Duncan et al., 2007; Jordan, Glutting, & Ramineni, 2010; Watts, Duncan, Siegler, & Davis-Kean, 2014). For example, researchers have found that beliefs such as task-related academic motivation and self-concepts about performance have reciprocal and cumulative effects on future math achievement (Aunola, Leskinen, & Nurmi, 2006; Marsh & Martin, 2011). In particular, beliefs about learning mathematics are generally associated with greater effort, higher self-efficacy in mathematics, and engagement in mathematical learning (Pajares & Miller, 1994; Schoenfeld, 1989; Wigfield & Meece, 1988), which are related to higher math achievement. In our conceptual framework, children's beliefs about math utility include utility value (the perceived usefulness of math), and productive disposition (the belief that math is useful and worthwhile, and that effort in math pays off).

Eccles and colleagues (e.g., Eccles & Wigfield, 2002; Eccles, Wigfield, Harold, & Blumenfeld, 1993; Wigfield & Eccles, 2000) have extensively studied the link between motivation and achievement using their expectancy-value theory of motivation. Utility value is one of the two components (interest and utility value) of Eccles' subjective task value theory (Eccles & Wigfield, 2002; Eccles et al., 1993). The expectancy-value theory suggests that children's expectations of success and the value they place on academic tasks influence achievement choices, performance, effort, and persistence. This study extends previous work on the topic by considering children's knowledge and beliefs about math utility. Participants are elementary school age children, a younger age group than is typically studied. This study also explores relations between children's math utility conceptions and their engagement in math activities at home to better understand how children's activities outside of the school context may be associated with their math utility conceptions.

Utility value. The majority of research examining the link between motivation and academic achievement uses Eccles' expectancy-value theory of motivation. The body of work related to the expectancy-value theory demonstrates that subjective task value, the value that one assigns to a task, including utility value, is positively related to math achievement test scores, grades in math, and the number and type of upper-level math courses selected in higher school (Guo, Marsh, Parker, Morin, & Yeung, 2015; Marsh & Martin, 2011; Marsh et al., 2005; Musu-Gillette et al., 2015; Singh, Granville, & Dika, 2002). This is particularly important, because subjective task value tends to be relatively high in early-to-late elementary school grades but declines significantly beginning around the transition to middle school (Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002). Understanding how to maintain more positive utility value beliefs is important, because positive subjective task value beliefs are associated with higher math achievement, lower math anxiety, and higher-level math course selection for fourth through ninth grade children (Meece, Wigfield, & Eccles, 1990; Spinath, Spinath, Harlaar, & Plomin, 2006).

The current study extends Eccles' subjective task value theory by expanding the ways in which utility value is measured with children. We focus on utility value because it is the only component within Eccles' theoretical model that specifically relates to math utility conceptions and achievement (see also Mazzocco et al., 2012).

Productive disposition. The National Research Council (2001) recognized the importance of math utility by including productive disposition, children's beliefs that they are users of math, and that math is useful and worthwhile, as one of their five "strands" of math proficiency. A productive disposition towards math is important for developing math knowledge and skills (Clements, 2001; Muis, 2004). The limited research on children's beliefs about how effort and engagement in math will benefit their math skills focuses on beliefs about mathematical learning and problem solving (De Corte, Op't Eynde, & Verschaffel, 2002; McLeod, 1992; Schoenfeld, 1992; Tsao, 2004) and has primarily used older children and adolescents.

Developmental Changes in Math Utility Conceptions

As children progress through school, their knowledge of math concepts changes (Clements & Sarama, 2014; Geary, 2006; Rittle-Johnson, 2017). They learn new mathematical operations and procedures and are exposed to new types of problems. With new exposures to math in their environment, children have the potential to build new knowledge about the usefulness of math in daily activities and different sources from which they can learn math (Clements & Sarama, 2007; Papadakis et al., 2017; Permutter et al., 1997). Little research has examined grade-level differences in children's definitions of what math is and whether they believe it is learned or used outside of the school context. However, research in other related math conceptions shows the importance of examining developmental changes.

Several studies have shown that competence/expectancy beliefs in math decline from elementary school through high school (Dweck & Elliott, 1983; Jacobs et al., 2002; King & McInerney, 2014; Muenks et al., 2018; Nagy et al., 2010). Less is known about the development of math utility beliefs. Musu-Gillette et al. (2015) found that, on average, utility value was highest in fourth grade; children showed an overall decline in math utility value through early college, although the rates of decline reflected group differences. However, other research suggests that math utility beliefs can also improve through targeted interventions (Hulleman et al., 2017; Jansen, 2012; Mitchell, 1999). Nevertheless, the youngest children in these studies were in fourth grade, so this research does not offer information about developmental changes in early elementary school. The present study builds on prior research by examining grade-level differences in knowledge and beliefs about math utility for early elementary-age children.

Math Engagement at Home

Children acquire mathematics knowledge from their environment even before they start school (e.g., Clements & Sarama, 2014; Elliott & Bachman, 2017; Ginsburg, Lee, & Boyd, 2008; Siegler & Mu, 2008). Nevertheless, children's engagement in math is limited (Plewis, Mooney, & Creeser, 1990; Saxe, Guberman, & Gearhart, 1987). For example, Tudge and Doucet (2004) found that preschool age children from Black and White, low- and middle- socioeconomic status families infrequently engaged in math-related activities either at home or at their child care centers. Moreover, even though children may engage in activities that have the potential to foster math skills, they do not necessarily focus on math when engaging in these activities. For example, even when a child plays with blocks, something which could involve math, s/he may focus on the color or texture of the blocks rather than the shape or number, two potential math-related components. In addition, other research has shown that even though children may engage in math activities, they are likely to be involved in basic math. Seo and Ginsburg (2004), for example, observed young children during free play and found that children engaged in math-related talk and activities, but the complexity of their interactions was often low.

Children's limited engagement in math at home is problematic given the relevance of this involvement for fostering math learning. Engagement in developmentally appropriate math activities at home, is generally positively associated with children's early math knowledge, especially for children in kindergarten and early elementary school (see reviews by Blevins-Knabe, 2016; Elliott & Bachman, 2017; Thompson, Napoli, & Purpura, 2017). Engagement in formal math activities, like completing worksheets, and informal ones, like playing board games, positively predicts children's math skills (LeFevre, Polyzoi, Skwarchuk, Fast, & Sowinski, 2010; LeFevre et al., 2009; Niklas & Schneider, 2014; Ramani & Siegler, 2008, 2014; Skwarchuk, Sowinski, & LeFevre, 2014). In addition, the frequency and quality of parents' "number talk" relates to children's development of early number skills (Gunderson & Levine, 2011). For example, Levine and colleagues (Gunderson & Levine 2011; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010) found that the amount of talk about number that parents engaged in significantly predicted children's later cardinal-number knowledge. Although the majority of parents' math talk was labeling cardinal values or counting, talk about more advanced content, like large number sets (4-10) with corresponding objects present, was most strongly associated with children's subsequent cardinal number knowledge. Whereas some research has examined which math activities children engage in at home (LeFevre et al., 2009; 2010; Saxe et al., 1987; Sonnenschein, et al., 2012; Sonnenschein, Metzger, & Thompson, 2016), little research has described how children engage in such activities. This is important because, as noted previously, whether an activity fosters math skills may depend upon the nature of engagement in that activity.

Relation between Children's Math Utility Conceptions and Engagement

Limited research has examined the relation between children's math utility conceptions, as defined in the current study, and their engagement in math activities at home. Most of the research examining these constructs has focused on one dimension of math conceptions (Eccles et al., 1993; Jacobs et al., 2002) or has examined the association between home engagement and achievement (LeFevre et al., 2010; Levine et al., 2010). The limited research that has examined math conceptions and engagement in math activities at home has shown that the quality of the home numeracy environment, indexed by measures of cognitive stimulation and home learning opportunities, positively predicts children's interest, a different aspect of subjective task value, for engaging in math activities (Gottfried, Fleming, & Gottfried, 1998). Similarly, Eccles' expectancy-value theory of motivation (Eccles & Wigfield, 2002) posits that parents' beliefs about math utility and the support they provide to their children with math also contribute to children's utility value beliefs (Chouinard, Karsenti, & Roy, 2007; Simpkins, Fredricks, & Eccles, 2012; Wigfield & Eccles, 2000).

The Present Study

This study examines rising first through fourth grade children's math utility conceptions (knowledge about math, beliefs about math utility), their engagement in math at home, and the association between the two.

1) What knowledge about math do children have and how does this differ across grade levels? We examine how they define math (math concepts), and the extent to which they are aware of the potential uses of math outside of school (applicability). Based on prior research examining children's definitions of math (Mazzocco et al., 2012; Perlmutter et al., 1997), we hypothesize that children's definitions of math will focus primarily on basic content knowledge, like numbers and operations. We also hypothesize that children will think math is something used primarily in school (Perlmutter et al., 1997). As children complete more grade levels in school and have more experience with math in and outside of school, their knowledge about math will shift to include higher-level operations, such as multiplication/division and skip counting (e.g., counting by 2s or 5s), and they may learn and experience more applications of math in daily life.

2) How much utility value do children place on math tasks (utility value) and who do they see as users of math (productive disposition)? Do these beliefs differ across grade levels? Eccles et al.'s (1993) work suggests that young children place high value on math utility. Current

research does not inform hypotheses regarding young children's productive disposition, so this study explores the descriptive nature of children's math utility beliefs.

3) What kinds of math activities do children report engaging in most frequently at home? Does frequency of engagement vary by grade level? Additionally, do children identify math-related aspects of their engagement in some activities? There is limited research on math home engagement to inform hypotheses regarding the frequency and nature of children's math home engagement. Accordingly, this study explores the descriptive nature of children's math home engagement.

4) What is the association between children's math utility conceptions and their home math engagement? Based on previous research that finds associations between math engagement and children's math skills (e.g., LeFevre et al., 2010; Siegler & Ramani, 2009; Skwarchuk et al., 2014), we hypothesize a similar relation exists between math engagement and children's conceptions, such that the frequency of children's home math engagement is positively associated with children's knowledge about math (math applicability) and beliefs (utility value).

The present study extends prior research in three ways. One, it adds to the very limited research on children's knowledge of how math is used in real-world contexts and how that knowledge relates to math engagement. Two, it examines productive disposition, an understudied construct, in children in first through fourth grade, an understudied age group. Three, this study investigates associations between math conceptions and children's engagement in math activities at home, something we know little about because research on children's engagement has focused primarily on the association between engagement and math skills.

Method

Participants

Ninety-nine children (58 boys) were recruited during the summer and early fall of 2010 and 2013 from schools and summer camps in the mid-Atlantic region. Most of the children (82%) were interviewed during the summer. Thirty-three participating children (Mean age = 6.43 years, *SD* = 0.39) were entering or had just entered first grade, 23 second grade (Mean age = 7.33 years, *SD* = 0.35), 23 third grade (Mean age = 8.42 years, *SD* = 0.33), and 20 fourth grade (Mean age = 9.51 years, *SD* = 0.42). Children were European American/White (*n* = 48), African American/Black (*n* = 21), Chinese American (*n* = 10), Hispanic/Latino (*n* = 10), or multiracial (*n* = 10). Five of the Hispanic/Latino children spoke primarily Spanish; the remaining five spoke primarily or only English. We did not collect specific data regarding parents' highest education level or household income, but we know that the majority of our sample was recruited from locations that serve middle income families whose parents, on average, have at least a Bachelor's degree. However, about 20% of our sample was recruited from locations that serve low income families, whose parents, on average, have not completed a college degree.

Measures

Knowledge about math.

Math concepts. Children were asked "What is math?" consistent with questions from Perlmutter and colleagues (1997). Coding of responses were based on the NCTM (2000) content and process standards (i.e., numbers and operations, problem solving). See Table 1 for a list of codes and exemplary quotes for all constructs. For this and other open-ended responses, inter-rater reliability was established by having two raters independently code up to 50% of the responses for each item. The researchers met after coding the transcripts to review their codes and reached consensus. Inter-coder reliability was tested using Cohen's kappa (Cohen, 1960). A kappa guideline of .70 was used to determine acceptable inter-rater reliability (Fleiss, 1981; Landis & Koch, 1977). If acceptable kappa levels were not reached in the first round of coding, the coding scheme was reviewed and modified, if necessary, and a new set of responses was coded. This process continued until kappas were at least .70 for every coding category. Final kappas ranged from .70 to 1.00 for each code within each construct unless otherwise noted. Remaining responses were then coded by one of the raters who had reached acceptable reliability.

Applicability of math. We used three open-ended questions adapted from Perlmutter et al. (1997) to examine this construct (see Table 1 for a description of codes). The first question measured how children believe math knowledge is acquired. Children were asked "How do you learn math?"

Children also were asked, "How does {person[s] mentioned} use math?" for each specific person that the child first mentioned used math (this was a follow-up question which is discussed further under productive disposition: "Who uses math?"

The third question asked whether and how children believed that mathematics was used in 10 activities: playing board games, card games, and video games, doing puzzles, cooking, helping at the grocery store, building with blocks or Legos, using or playing with money, using maps or a globe, and keeping score in games or sports. Children first were asked, "Some children think math is used when they [play board games], some think math is not used at all. Do you think math is used when you play board games?" If children responded "yes," they were then asked how math was used in the activity. A child's response was coded on a 4 point scale: 0 if s/he did not identify that math was used or if s/he said that math was used but the description was not related to math (e.g., "*when cooking, you read the words on the page*"); 1 if the child said that math was used in the activity, but did not elaborate or articulate about how; 2 if s/he described a basic math skill; 3 if s/he described an advanced math skill. Inter-rater reliability was established by having two raters independently code about 50% of the responses. Because of the meaningful differences between scale values, we wanted to be sure that independent coders were in complete agreement before moving forward; therefore, 100% exact agreement was reached before coding the remaining responses. A composite was created by averaging scores for each activity examined. Cronbach's alpha for the applicability scale was .84.

Beliefs about math utility.

Utility value. We used six items to create a utility value measure (see Table 1). Items were adapted from measures used to grasp mathematics or reading motivation (Baker & Scher, 2002; Eccles et al., 1993; Sonnenschein, Baker, & Garrett, 2004; Wigfield & Guthrie, 1997). Example of items include "Math is useful outside of school" and "It is important for me to learn math." Children were asked to report whether they felt each item was "not at all like me," "a little like me," or "a lot like me." Three non-math activities were presented as examples at the outset in order to familiarize children with the rating scale. A composite was created by averaging the scores on the six items. Cronbach's for the utility value measure was .69.

Productive disposition. To measure the extent to which children see themselves and others as users of math, we used Perlmutter et al.'s (1997) question, "Who uses math?" (see Table 1). Responses were categorized as teachers, parents, children, and other adults (most commonly mentioned other adults were scientists, architects, accountants, and adult relatives). An additional category was coded if a child said that everyone does math. Final kappas for each coding category were 1.00.

Math engagement at home. An index for child-reported frequency of math home engagement was created by averaging frequency of engagement in 13 mathematics activities at home, including playing board games, playing video games, helping with cooking, helping at the grocery store, and building with blocks or Legos (see Table 1). These items were adapted from other measures of children's frequency of math engagement at home (Sonnenschein et al., 2012). Response options were "almost never," "sometimes," and "almost every day." Three non-math activities were presented as examples to familiarize children with the rating scale. Based on results from pilot testing, the rising first graders received an abbreviated version of the math engagement measure. It did not include four activities: keeping score in games or sports, playing with or using money, using maps or globes, and using a calculator. Cronbach's alpha for the frequency of engagement scale was .66 for all 13 items and .50 for the 9 items common to all children. The less than optimal alpha values likely reflect that a child's engagement in one activity does not necessarily mean s/he will engage in another activity.

Demographic information. As part of the consent documents, parents were asked to provide their child's age, grade level in school in the fall, gender, and race/ethnicity (African American/Black, European American/White, Hispanic/Latino, Asian/Pacific Islander, or "Other").

Procedure

Children were interviewed individually by a trained graduate or an advanced undergraduate research assistant. Each interview took place in an empty room in the child's home or summer camp/school. Sessions lasted 15-20 minutes and were recorded. The interviewer also took notes of the child's responses. Children were interviewed in their preferred language which was English for all but five of the Latino/a children. Those five children were interviewed in Spanish by a native Spanish speaker. Interviews conducted in Spanish were transcribed in Spanish, then translated into English, and then back-translated to ensure accuracy.

Analytic Plan

The math utility conceptions model presented in this paper (see Figure 1) is the conceptual model that guides this study. We examine the components individually and some relations among them, but do not statistically test the model itself. We use a quantitative approach, described within each of the results subsections, to address the research questions. We complement these quantitative findings, as appropriate, with illustrative quotes from participants.

Length of utterance. We completed a length of utterance analysis for each open-ended item to control for potential developmental differences in the length of children's responses as well as the possibility that children who simply speak more words may articulate more about their math conceptions. Similar to how length of open-ended responses was assessed in related research (e.g., Denscombe, 2008; Wang, 2004), for each response, we counted the number of words the child used, with the exception of filler utterances such as "um" and "uh."

Preliminary analyses showed that there were significant length of utterance differences across grade levels for some items. Accordingly, for analyses examining differences between each grade-level for open-ended item responses, we controlled for children's length of utterance for that response. We conducted analyses with and without controlling for length of utterance; however, the pattern of results was very similar. In what follows, we only report analyses controlling for length of utterance.

Results

Math Utility Conceptions

Knowledge about math. Analyses for open-ended items of math concepts and applicability of math were coded and analyzed descriptively. Depending on the nature of the dependent variable, dichotomous or continuous, we used logistic regressions or ANCOVAs to examine grade-level differences in responses. ANCOVAs with grade level as the between subjects factor and length of utterance as the covariate were used to examine grade-level differences in scale scores and number of activities that children identified as featuring math. Fisher's Least Significant Difference (LSD) post-hoc tests were used to examine differences between specific grade levels.

Math concepts. As hypothesized, children's definitions of math indicated a view of math that was heavily focused on numbers and operations (see Table 2). Ninety percent of children defined math as some form of numbers and operations. Most children stated that math is calculations (67%, number transformations like addition and multiplication) and counting (18%). For example, children often gave responses like, "*[Math is] something when you learn about numbers and how to add them up*" or they gave more elaborate descriptions of operations, "*[Math is] something that you learn about numbers and…when the teacher says two plus two, you say it's four and four and four equals and she snaps her fingers and we will all say eight.*"

A few children (12%**)** mentioned math processes, including problem solving ("*you use math to figure out difficult problems,"*) and connections ("*[Math is] something to help you go along the way because math is in a lot of things, in science, geometry, even art" or "Math is this thing with numbers and like everything, and when I say everything, I mean everything, involves math,"*). Children mentioned a mean of 1.33 different categories of math (*SD* = 0.77; Range = 0-

4), which indicates that, on average, children's knowledge of the breadth of math concepts is somewhat limited.

As hypothesized, there was a difference across grade levels in children's knowledge of math concepts. Controlling for length of utterance, the odds that children mentioned calculations (number transformations), $\beta = 0.71$, $OR = 2.03$, $p = .001$, approximately doubled for each grade-level increase. This suggests that children may define math by what they are doing in school, given that younger children's number transformations were typically addition and subtraction and older children's were multiplication and division. There were not significant differences across grade level for other coding categories within math concepts.

***Applicability of math*.** As hypothesized, children viewed math as something that is learned and used in school (see Table 3). When asked how they learn math, children mentioned school (74%) and learning from teachers (55%) more often than learning from parents (27%) or non-school related activities (12%). Sixty-four percent of children mentioned only one way to learn math; 28% mentioned more than one way. For example, two different responses coded as learning at school were *"I learned it from school when I was in first grade."* and *"…you have to go to school to learn math."* Two responses coded as learning math from teachers were, "*well the teacher teaches us the subject and we have to write it down…we get a quiz to do all the things like if I was on multiplication that had to put multiplication facts*" and "*teachers show us like how to add and subtract.*"

As hypothesized, there were differences across grade levels in children's knowledge of the ways in which math can be acquired. Controlling for length of utterance, the odds that children mentioned that math is learned in school, $\beta = 1.00$, $OR = 2.72$, $p < .001$, and that math is acquired with the help of a teacher, $\beta = 0.43$, $OR = 1.53$, $p = .026$, increased with each additional grade level. Also, the odds that children mentioned friends or siblings (generally in the context of helping with school work) increased with each additional grade level, $\beta = 1.55$, $OR = 4.71$, $p = .047$.

When asked the ways in which different people use math, children primarily mentioned math operations (62%, addition/subtraction, multiplication/division, etc.) and school-related uses of math (35%) rather than home-related uses (< 1%; see Table 4). Of the children who mentioned teachers as people who use math ($n = 43$), 76% reported that teachers primarily use math in the school context for teaching children rather than in their daily lives outside of school (*"...to teach their students to learn"* or "*…well she does let us make a one hundred chart and she lets us use digi blocks"*). In contrast, children were aware of parents (33% of $n = 18$) and other adults (70% of $n = 38$) typically using math for job-related activities (see Table 5). For example, one children shared, *"[My parents use math] at work"* or *"...she helps people like if they want to go on a cruise then she has to like take this much money and add it up to this much money"*, Another child mentioned, *"...scientists use it for like chemicals and stuff like ¼ of something"* or *"...cooks measure things out,"*)

Children's awareness of math in daily activities was measured by how well children were able to identify and articulate the math potential in several every day activities. On average, children were able to identify that math was used in a specific activity but did not or could not fully describe how it was used ($M = 1.26$, $SD = 0.79$; Range = 0.0-2.90 out of possible 3). As hypothesized, controlling for length of utterance, children's knowledge of the applicability of math in daily activities increased significantly across grade level, $F(3,94) = 13.25$, $p < .001$, partial $\eta^2 = .297$. Third ($M = 1.84$) and fourth graders' knowledge ($M = 1.90$) did not differ significantly, $p = .918$. Both third and fourth graders had significantly higher math awareness scores than first (M = 0.73, p < .001, < .001, respectively) and second graders (M = 0.90, p < .001 < .001, respectively) who did not differ from each other, p = 285.

Although, on average, children's knowledge of the applicability of math was limited, most children were able to articulate their awareness of how math is used in some individual activities but not others. Typical examples of children identifying math with a basic math concept (score of 2) included "*Well sometimes in dice games, you need to count the number on the dice*" or "*The cards have numbers on them*" or "*When you check out…you gotta count the money.*" Typical examples of children identifying math with an advanced math concept (score of 3) included "*…fractions like…half a cup of sugar or a quart of water*" or "*If our team had 7 and the other team had like 3, then our team would have four more points*" or "*[about playing with puzzles] You have to get the right pieces in the perfect size where it has to be.*"

Beliefs about math. For utility value, descriptive statistics are presented, and ANOVAs used to examine grade-level differences. We did not control for length of utterance in these comparisons, because they were scale rather than open-ended items. For productive disposition, descriptive analyses are presented, and logistic regressions used to determine grade-level differences in responses to each coding category. Because of grade-level differences, relations between the math awareness and utility value scores were examined using partial correlations, controlling for grade level.

Utility value. As hypothesized, in general, children believe that math is useful (M = 2.58, SD = 0.41, on a scale of 1-3). Utility value scores differed across grade level, $F(3,95) = 4.15$, p = .008, partial $\eta^2 = .116$. Third (M = 2.70) and fourth graders' (M = 2.76) usefulness scores were comparable to each other, p = .600, and significantly higher than first graders' (M = 2.41), p =

.009, .002, respectively. First and second graders' (M = 2.54) scores did not differ significantly, p = 252.

***Productive disposition*.** Consistent with children's conceptions that math is primarily school-based, children viewed math as used primarily by their teachers or their peers, rather than their parents or themselves (see Table 6). When asked who uses math, a higher percentage of children mentioned children/classmates/siblings (53%), teachers (43%), and other adults (38%) than parents (18%).Controlling for length of utterance, for each additional grade level, the odds of children mentioning that other adults (e.g., scientists, engineers, architects, $\beta = 0.54$, OR = 1.71, $p = .007$) use math or that everyone, $\beta = 0.54$, $OR = 1.72$, $p = .035$, uses math increased. There were no significant age differences for any other category.

Controlling for children's grade level, children's knowledge about math applicability was significantly related to their math utility scores, $r(94) = .28$, $p = .005$. The more aware children are that math features into their daily activities, the more strongly they believe that math is useful and important.

Home Math Engagement

Descriptive analyses for the frequency scale of children's home math engagement categories are presented. Because of grade-level differences in math utility conceptions, partial correlations, controlling for grade level, examined whether home engagement was associated with math applicability and math utility scores.

Children, on average, reported "sometimes" engaging in math activities at home (M = 1.84, SD = 0.30; Range = 1.38-2.92). Thirty-five percent of children reported engaging in math activities at home almost every day, 50% reported engaging sometimes. The activities which children reported engaging in most frequently were using a computer (M = 2.11 out of 3), playing video games (*M* = 2.09), keeping score in games (*M* = 2.09), building with blocks or Legos (*M* = 2.05), and helping at the grocery store (*M* = 2.04). The activities which children reported engaging in least frequently were using maps or globes (*M* = 1.41), using a calculator (*M* = 1.50), and playing with or using money (*M* = 1.69). Mean frequency of engagement did not differ significantly across grade level, $F(3, 95) = 2.07$, $p = .109$.

Association between children's math utility conceptions and home math engagement. To determine how children's math utility conceptions were associated with children's engagement in home-based math activities, we examined whether components of math utility conceptions were related to the frequency of math engagement. Contrary to our hypothesis, after controlling for grade level and length of utterance, children's overall frequency of engagement was not significantly associated with their overall math awareness scale scores, $r(93) = -.13$, $p = .215$. Also contrary to our hypothesis, frequency of engagement was not significantly associated with children's math utility scores, $r(97) = -.03$, $p = .796$, or any individual item within the utility value scale, $p > .05$.

We examined, for each activity, whether frequency of engagement was related to awareness of math scores for that activity, controlling for grade level and length of utterance. Again, contrary to our hypothesis, after controlling for grade level and length of utterance, children's engagement in a specific activity was not associated with their awareness of math in that activity, $p > .05$, except for playing video games, $r(94) = .21$, $p = .038$. Finally, we examined whether children's awareness of math in one activity was related to their awareness of math in other activities. After controlling for grade level and length of utterance, children's awareness of math in a given activity was rarely significantly correlated (with a few exceptions) with awareness of math in other activities (see Table 7). The lack of significant correlations suggests that even when children are aware of math in one activity, they may not be able to generalize that knowledge to other activities as well.

Discussion

This study examined children's knowledge and beliefs about math and its utility, and the relation between such knowledge and beliefs and their engagement in math activities at home. This was one of the first studies to examine this topic with rising first through fourth graders. Understanding children's math utility conceptions is important for getting a more complete picture of their math knowledge and beliefs. Building this understanding is important, because research shows that children's knowledge and beliefs about math are associated with their math achievement (e.g., Hulleman et al., 2017; Mazzocco et al., 2012; Papadakis et al., 2017; Rittle-Johnson, 2017). Three findings were of particular interest.

One, consistent with Perlmutter et al. (1997) and Mazzocco et al., (2012), children's views about what math is were heavily focused on low-level math operations, such as counting and number transformations, and as something learned and used primarily in school. Most children conceptualized math as school-based; they displayed limited knowledge of how math features in their daily lives outside of school. In other words, they did not seem to associate the math they learned in school with the math they may use in their everyday activities outside of school. It could be that to foster connections between math learned at school and math embedded in their daily lives, these connections must be made explicit. School seems to be a natural place for explicit discussions about math to occur, as what children are doing at home does not appear to be sufficient for developing constructive conceptions about math utility. Helping children see the relations between school math and the math they are using in their own daily activities may be a way for them to develop a sense of themselves as math users. This is important because seeing oneself as a user of math is an integral part of developing math proficiency (National Research Council, 2001).

Two, there were differences in children’s knowledge of math across grade levels, even after controlling for what could be differences in aspects of language skills. Older children viewed math as more school-based than did younger children. Specifically, older children conceptualized math as number transformations that likely coincide with the higher-level operations they are learning in school and that math is learned mostly at school, with the help of teachers and classmates/peers. On the other hand, older children reported higher math utility value and were able to identify more ways in which people use math in their daily lives outside of the school. However, even though children’s math awareness increased with grade, older children were still often unable to identify how math is used in daily activities, indicating that there may be limitations to their knowledge of math applicability at this age. There are multiple factors that may contribute to these differences across grade levels. Children’s experiences at school and home provide them with increased math language and knowledge, which can improve the metacognitive skills needed for math development (Ginsburg et al., 2008). These experiences could include formal math lessons, exposure to adults who model math uses and/or discuss how they use math in their lives, and children’s own math use in their daily activities.

Three, regardless of child’s grade level, the frequency of engaging in math activities at home was not associated with knowledge about math applicability in those activities. Math home engagement was also not associated with beliefs about the utility of math. One reason may be that children are not labeling their activities as “math.” Labeling these activities as “math” is important, because research shows that children’s math language is associated with their ability to recognize and communicate about their math learning and well as their math achievement

(Bay-Williams & Livers, 2009; Purpura & Reid, 2016; Rothman & Cohen, 1989). Children's understanding of their own math learning facilitates their understanding of math concepts and connections (NCTM, 2000). Thus, it seems particularly important in the development of math utility conceptions that children be knowledgeable about the potential ways that math features into their daily activities and be able to apply the label of "math" to those activities. They may be less likely to consider it an activity in which math is useful if they do not provide such a label. Children may still learn math skills through engaging in math activities, but may be less likely to develop math utility conceptions from those activities if they do not label the activity as "math."

Another reason for the lack of relation between engagement and math conceptions may be because, although children were exposed to experiences in their homes that could enable them to acquire math skills (Ginsburg, Duch, Ertle, & Noble, 2012; Ginsburg et al., 2008; Sarama & Clements, 2006, 2007; Siegler & Mu, 2008), the nature of their engagement may not facilitate learning if they are not engaged in math-related aspects of such activities (Seo & Ginsburg, 2004). When we asked children to describe how they engaged in two math activities (helping with cooking and helping at the grocery store), we found that only 18% of children who helped with cooking and 8% who helped at the grocery store mentioned doing anything related to math while engaging in that activity. Most children reported non-math-related engagement when explaining what they do while cooking and helping at the grocery store including gathering ingredients ("*I get my mom the meat"* or "*getting the things my mom wants or the bags, putting stuff in bags"*), mixing/pouring ingredients ("*well, I help my mom to make soup and I put the soup in the saucepan and I wash the potatoes"*), and reading instructions or shopping lists ("*I read the words on the page"* or "*my mom hands me the list and then I read it so I know what to pick out"*). Thus, in spite of children reporting that they were frequently involved in these activities (55% helped with cooking and 77% at the grocery store), their participation may not be fostering math learning. Although the majority of children in this study reported engaging in activities that could foster math skills, few reported engaging in aspects of those activities that actually involved math. These findings suggest that parents and teachers may increase children's math knowledge by actively modeling math-related behaviors and/or math language, engaging their children in math-related aspects of common daily life activities (e.g., cooking, grocery shopping), and by making sure their children engage in a variety of math activities (Sonnenschein et al., 2016).

Implications for Practice

Eliminating math educational disadvantages is an important national priority in math education policy (National Mathematics Advisory Panel, 2008). Large-scale international tests, completed by fourth, eighth, and twelve graders, show deficits for children in the U.S. as early as fourth grade (U.S. Department of Education, 2013, 2015a, 2015b, 2018). The Standards for Mathematical Practice within the Common Core curriculum call for children to learn how to connect the math they are learning in school with math they need to solve everyday problems (CCSSI, 2010; Kendall, 2011). Teachers can do this by embedding problems in everyday situations and explicitly connecting math learning to daily activities.

Another avenue for improving children's math conceptions is focusing on home-based opportunities. Parents can demonstrate, by modeling or explicitly discussing with children, the ways in which they use math in their daily lives (e.g., paying bills, cooking, or counting money at the grocery store) in order for children to recognize their parents as users of math. The extent to which parents engage in number talk at home relates to children's number knowledge (Blevins-Knabe & Musin-Miller, 1996; Gunderson & Levine, 2011; LeFevre et al., 2009; Levine et al., 2010). Discussion of math utility value may have a similar impact on children's math utility conceptions. There also may be ways to add math utility to these informal discussions to increase children's knowledge about the applicability of math and develop positive beliefs about the usefulness of math. As Levine and colleagues (Gunderson & Levine, 2011; Levine et al., 2010) have shown with other aspects of math talk, parents' discussions of math are related to their children's knowledge. However, to help parents do this, we must better understand the ways in which their conceptions impact the nature of children's engagement in math activities at home. For high school children, providing parents with materials with information about the utility of math in STEM careers led to gains in parents' math utility value, the number of STEM-related courses that children chose in their junior and senior years of high school, and increased engagement in STEM-related career fields. (Harackiewicz, Rozek, Hulleman, & Hyde, 2012; Rozek, Svoboda, Harackiewicz, Hulleman, & Hyde, 2017). However, this type of intervention has not been done with elementary school children. If parents are given helpful support, gains such as these may be possible for elementary age children as well.

Limitations and Future Directions

Although the present study provided new information about how children define math and how they see math relating to their lives, there are several limitations to consider. The relatively small sample size prevented exploration of differences related to children's racial/ethnic group and parents' socioeconomic status and educational level. Relatedly, although Eccles et al. (1993) subjective task value scale, which was adapted for this study, was validated with a primarily European American/White sample, it has not been validated with ethnically diverse children. Another limitation is the timing of data collection. As noted, most of the children (82%) were interviewed during the summer months, when they typically have less exposure to academic instruction. This may have impacted how frequently they engaged in math activities at home as well as their concepts of when, where, and by whom math is used. Analyses comparing the responses of children interviewed in the summer with those in the fall showed a similar pattern of responses. And, most children's responses, regardless of the timing of data collection focused on school-related conceptions.

Perhaps the largest potential limitation is the inability to distinguish whether children lacked math utility concepts or just could not articulate well their conceptions. Clearly, children's verbal abilities increase with age (Berko Gleason & Ratner, 2012; MacWhinney, 2010). However, children as young as preschool have demonstrated the ability to use rating scales and to describe self-concepts about their academic abilities and learning in reliable and valid ways. For example, researchers successfully used rating scales with children in preschool through sixth grade to measure math self-concepts (Marsh, Ellis, & Craven, 2002), subjective task value (Eccles et al., 1993; Wigfield et al., 1997), frequency of engagement in home math activities (Ramani & Siegler, 2008), and child-teacher relationships (Li, Hughes, Kwok, & Hsu, 2012). The rating scales used in these research studies yielded acceptable to high reliability estimates with young children, were related to achievement and parent/teacher ratings, and showed consistent longitudinal patterns. Additionally, Mazzocco et al. (2012) asked children as young as second grade to describe their definitions of math. Children were able to provide responses that were related to their third grade math achievement. These examples of child-reported math self-concepts demonstrate young children's ability to report their beliefs about math. Finally, the coding schemes used in this study as well as the length of utterance analyses controlled for potential developmental differences in length and sophistication of responses. Given the wealth of evidence showing that children are reliable reporters, we think our results are valid. Our methods also gave children a voice in research that may impact their education (Grover, 2004).

Future research should explore the relation between children's math utility conceptions and math achievement. Mazzocco et al. (2012) found significant positive associations between second grade children's definitions of math and their number skills in third grade using the Calculation subtest from the Woodcock Johnson-Revised Tests of Achievement (Woodcock & Johnson, 1989). Additional research should focus on how children's math utility conceptions relate to a broader array of math skills (e.g., conceptual understanding, problem solving). Future research also should test the relations between the knowledge and belief about math utility conceptions components of the conceptual model used in this study.

Additionally, research has not examined the potential effects of home and classroom interventions on children's math conceptions. Documenting longitudinally how children's math utility conceptions develop naturally over time at home and in school is important. Research needs to explore how parents' and teachers' math utility conceptions impact the development of children's conceptions. Other home and school factors, such as amount of time children spend engaged in math learning in the classroom, the frequency with which parents help their children with math homework, or the extent to which teachers and parents label daily activities as "math" may be associated with the development of children's math utility conceptions. Such information could serve as the basis for interventions to improve children's math utility conceptions.

Conclusion

The primary goals of this study were to investigate children's math utility conceptions and understand whether children's engagement in math at home is associated with their math utility conceptions. Exploring children's conceptions about how math is used and by whom may help guide future interventions to improve math learning. By increasing young children's knowledge of applications of math outside the school context and beliefs about the usefulness of math, parents and educators can help to increase children's math proficiency.

References

Aunola, K., Leskinen, E., & Nurmi, J.-E. (2006). Developmental dynamics between mathematical performance, task motivation, and teachers' goals during transition to primary school. *British Journal of Educational Psychology, 76,* 21-40. doi:10.1348/000709905X51608

Baker, L., & Scher, D. (2002). Beginning reader's motivation for reading in relation to parental beliefs and home reading experiences. *Reading Psychology, 23*, 239-269. doi:10.1080/713775283

Barab, S.A., Squire, K.D., & Dueber, W. (2000). A co-evolutionary model for supporting the emergence of authenticity. *Educational Technology Research and Development, 48,* 37-62. doi:10.1007/BF02313400

Bay-Williams, J. M., & Livers, S. (2009). Supporting math vocabulary acquisition. *Teaching Children Mathematics, 16*, 238-245.

Berko Gleason, J. B., & Ratner, N. B. (2012). *The development of language*, 8th edition. Boston, MA: Pearson.

Blevens-Knabe, B. (2016) Early mathematical development: How the home environment matters. In B. Blevins-Knabe & A. M. B. Austin (Eds.), *Early Childhood Mathematics Skill Development in the Home Environment* (pp. 7-28). Switzerland: Springer International Publishing. doi:10.1007/978-3-319-43974-7_2

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting, 5,* 35-45. doi:10.1002/(SICI)1099-0917(199603)5:1<35::AID-EDP113>3.0.CO;2-0

Browning, C. A., Harrison, J. L., Harrison, R., Ko, Y. Y. W., Panorkou, N., & Yoder, M. A. (2016). Using children's thinking to develop mathematical content knowledge. In L. C. Hart, S. Oesterle, S. S. Auslander, & A. Kajander (Eds.), *The Mathematics Education of Elementary Teachers: Issues and Strategies for Content Courses* (pp. 25-50). Charlotte, NC: Information Age Publishing, Inc.

Chouinard, R., Karsenti, T., & Roy, N. (2007). Relations among competence beliefs, utility value, achievement goals, and effort in mathematics. *British Journal of Educational Psychology*, *77,* 501-517. doi:10.1348/000709906X133589

Clark, K. E. (1988). The importance of developing leadership potential of youth with talent in mathematics and science. In J. Dreyden, S. A. Gallagher, G. E. Stanley, & R. N. Sawyer (Eds.), *Report to the National Science Foundation: Talent Identification Program/National Science Foundation Conference on Academic Talent* (pp. 95–104). Durham, NC: National Science Foundation.

Clements, D.H. (2001). Mathematics in the preschool. *Teaching Children Mathematics, 7*, 270-275.

Clements, D. H., & Sarama, J. (2007). Early childhood mathematics learning. In F. K. Lester Jr. (Vol. Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics, Vol. 1* (pp.461-566). Charlotte, NC: Information Age Publishing.

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach (2nd ed.).* New York: Routledge.

Cohen, J. (1960). A coefficient of agreement for nominal scales. *Educational and Psychological Measurement, 20,* 37-46.

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

De Corte, E., Op't Eynde, P., & Verschaffel, L. (2002). "Knowing what to believe": The relevance of students' mathematical beliefs for mathematics education. In B. K. Hofer & P. R. Pintrich (Eds.), *Personal epistemology: The psychology of beliefs about knowledge and knowing* (pp.297-320). Mahwah, NJ: Lawrence Erlbaum Associates.

De Corte, E., & Verschaffel, L. (2006). Mathematical thinking and learning. In W. Damon & R. M. Lerner (Series Eds.) & K. A. Renninger & I. E. Sigel (Vol. Eds.), *Handbook of Child Psychology: Vol. 4 Child psychology in practice* (6th ed., pp. 103-152). New York: Wiley.

Denscombe, M. (2008). The length of responses to open-ended questions: A comparison of online and paper questionnaires in terms of a mode effect. *Social Science Computer Review, 26,* 359-368. doi:10.1177/0894439307309671

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., … Japel, C. (2007). School readiness and later achievement. *Developmental Psychology, 43,* 1428-1446. doi:10.1037/0012-1649.43.6.1428.supp

Dunlosky, J., & Rawson, K. A. (2012). Overconfidence produces underachievement: Inaccurate self evaluations undermine students' learning and retention. *Learning and Instruction, 22,* 271-280. doi:10.1016/j.learninstruc.2011.08.003

Dweck, C. S., & Elliott, E. S. (1983). Achievement motivation. In E. M. Hetherington (Ed.), *Socialization, personality, and social development.* New York: Wiley.

Eccles, J. S., & Wigfield, A. (2002). Motivational beliefs, values, and goals. *Annual Review of Psychology, 53,* 109-132. doi:10.1146/annurev.psych.53.100901.135153

Eccles, J. S., Wigfield, A., Harold, R., & Blumenfeld, P. (1993). Age and gender differences in children's self-and task perceptions during elementary school. *Child Development, 64,* 830-847. doi:10.1111/j.1467-8624.1993.tb02946.x

Elliott, L. E., & Bachman, H. J. (2017). How do parents foster young children's math skills? *Child Development Perspectives*, 6 pages. doi: 10.1111/cdep.12249

Fleiss, J. L. (1981). *Statistical methods for rates and proportions (2nd ed.).* New York: John Wiley.

Geary, D. C. (2006). Development of mathematical understanding. In W. Damon & R. M. Lerner (Series Eds.) & D. Kuhn & R. Siegler (Vol. Eds.), *Handbook of child psychology: Vol. 2 Cognition, perception, and language* (6th ed., pp. 777-810). New York: Wiley.

Ginsburg, H. P., Duch, H., Ertle, B., & Noble, K. G. (2012). How can parents help their children learn math?. B. H. Wasik (Ed.), *Handbook of Family Literacy* (2nd ed., pp. 51-65). New York: Routledge.

Ginsburg, H. P., Lee, J. S., Boyd, J. S. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report, 22,* 3-22.

Gottfried, A. E., Fleming, J. S., & Gottfried, A. W. (1998). Role of cognitively stimulating home environment in children's academic intrinsic motivation: A longitudinal study. *Child development, 69,* 1448-1460. doi:10.2307/1132277

Grover, S. (2004). Why won't they listen to us? On giving power and voice to children participating in social research. *Childhood, 11*, 81-93. doi:10.1177/0907568204040186

Gunderson, E. A., & Levine, S. C. (2011). Some types of parent number talk count more than others: Relations between parents' input and children's cardinal-number knowledge. *Developmental Science*, *14*, 1021-1032. doi:10.1111/j.1467-7687.2011.01050.x

Guo, J., Marsh, H. W., Parker, P. D., Morin, A. J., & Yeung, A. S. (2015). Expectancy-value in mathematics, gender and socioeconomic background as predictors of achievement and aspirations: A multi-cohort study. *Learning and Individual Differences*, *37*, 161-168. doi:10.1016/j.lindif.2015.01.008

Harackiewicz, J. M., Rozek, C. S., Hulleman, C. S., & Hyde, J. S. (2012). Helping parents to motivate adolescents in mathematics and science: An experimental test of a utility-value intervention. *Psychological Science, 23,* 899-906. doi:10.1177/0956797611435530

Herrington, J., Reeves, T. C., & Oliver, R. (2013). Authentic learning environments. In J.M. Spector, M.D. Merrill, J. Elen, & M. J. Bishop (Eds.), *Handbook of Research on Educational Communications and Technology* (pp. 401-412). New York: Springer.

Hulleman, C. S., & Harackiewicz, J. M. (2009). Promoting interest and performance in high school science classes. S*cience, 326*, 1410-1412. doi:10.1126/science.1177067

Hulleman, C. S., Kosovich, J. J., Barron, K. E., & Daniel, D. B. (2017). Making connections: Replicating and extending the utility value intervention in the classroom. *Journal of Educational Psychology*, *109*, 387. doi:10.1037/edu0000146

Jacobs, J. E., Lanza, S., Osgood, D. W., Eccles, J. S., & Wigfield, A. (2002). Changes in children's self-competence and values: Gender and domain differences across grades one through twelve. *Child Development*, *73*, 509-527. doi:10.1111/1467-8624.00421

Jansen, A. (2012). Developing productive dispositions during small-group work in two sixth-grade mathematics classrooms: Teachers' facilitation effort and students' self-reported benefits. *Middle Grades Research Journal, 7,* 37-56.

Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences*, *20*, 82-88. doi:10.1016/j.lindif.2009.07.004

Kendall, J. S. (2011). *Understanding Common Core State Standards*. Alexandria, VA: ASCD.

King, R. B., & McInerney, D. M. (2014). Mapping changes in students' English and math self-concepts: a latent growth model study. *Educational Psychology*. doi:10.1080/01443410.2014.909009

Krawec, J., Huang, J., Montague, M., Kressler, B., & de Alba, A. M. (2013). The effects of cognitive strategy instruction on knowledge of math problem-solving processes of middle school students with learning disabilities. *Learning Disability Quarterly, 36*, 80-92. doi:10.1177/0731948712463368

Landis, J. R. & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics, 33*, 159–174.

LeFevre, J.-A., Polyzoi, E., Skwarchuk, S.-L., Fast, L., & Sowinski, C. (2010). Do home numeracy and literacy practices of Greek and Canadian parents predict the numeracy skills of kindergarten children? *International Journal of Early Years Education, 18*, 55-70. doi:10.1080/09669761003693926

LeFevre, J.-A., Skwarchuk, S.-L., Smith-Chant, B.L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children's math performance in the early school years. *Canadian Journal of Behavioral Science, 41,* 55-66. doi:10.1037/a0014532

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology*, *46*, 1309-1319. doi:10.1037/a0014532

Li, Y., Hughes, J. N., Kwok, O. M., & Hsu, H. Y. (2012). Evidence of convergent and discriminant validity of child, teacher, and peer reports of teacher–student support. *Psychological Assessment*, *24*, 54-65. doi:10.1037/a0024481

MacWhinney, B. (2010). Language development. In M. H. Bornstein & M. E. Lamb (Eds.), *Developmental science: An advanced textbook* (pp. 389-424). NY: Psychology Press.

Marsh, H. W., Ellis, L. A., & Craven, R. G. (2002). How do preschool children feel about themselves? Unraveling measurement and multidimensional self-concept structure. *Developmental psychology*, *38*, 376-393. doi:10.1037//0012-1649.38.3.376

Marsh, H. W., & Martin, A. J. (2011). Academic self-concept and academic achievement: Relations and causal ordering. *British Journal of Educational Psychology*, *81*, 59-77. doi:10.1348/000709910X503501

Marsh, H. W., Trautwein, U., Lüdtke, O., Köller, O., & Baumert, J. (2005). Academic self-concept, interest, grades, and standardized test scores: Reciprocal effects models of causal ordering. *Child Development*, *76*, 397-416. doi:10.1111/j.1467-8624.2005.00853.x

Mazzocco, M. M., Hanich, L. B., & Noeder, M. M. (2012). Primary school age students' spontaneous comments about math reveal emerging dispositions linked to later mathematics achievement. *Child Development Research*, *2012*, doi:10.1155/2012/170310.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A* *project of the national council of teachers of mathematics* (pp. 575-596). New York: Macmillan Publishing Company.

Meece, J. L., Wigfield, A., & Eccles, J. S. (1990). Predictors of math anxiety and its influence on young adolescents' course enrollment intentions and performance in mathematics. *Journal of Educational Psychology*, *82*, 60-70.

Mitchell, T. (1999). Changing student attitudes towards mathematics. *Primary Educator, 5*(4), 2-7.

Muenks, K., Wigfield, A., & Eccles, J. S. (2018). I can do this! The development and calibration of children's expectations for success and competence beliefs. *Developmental Review*. doi:10.1016/j.dr.2018.04.001

Muis, K. R. (2004). Personal epistemology and mathematics: A critical review and synthesis of research. *Review of Educational Research, 74,* 317-377. doi:10.3012/00346543074003317

Musu-Gillette, L. E., Wigfield, A., Harring, J. R., & Eccles, J. S. (2015). Trajectories of change in students' self-concepts of ability and values in math and college major choice. *Educational Research and Evaluation*, *21*, 343-370. doi:10.1080/13803611.2015.1057161

Nagy, G., Watt, H. M., Eccles, J. S., Trautwein, U., Lüdtke, O., & Baumert, J. (2010). The development of students' mathematics self-concept in relation to gender: Different countries, different trajectories? *Journal of Research on Adolescence*, *20*, 482-506. doi:10.1111/j.1532-7795.2010.00644.x

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences of Education. Washington, DC: National Academy Press.

National Mathematics Advisory Panel (2008). Foundations for success: The final report of the National Mathematics Advisory Panel.

Niklas, F., & Schneider, W. (2014). Casting the die before the die is cast: The importance of the home numeracy environment for preschool children. *European Journal of Psychology of Education*, 29, 327–345. doi:10.1007/s10212-013-0201-6

Pajares, F., & Miller, M. D. (1994). Role of self-efficacy and self-concept beliefs in mathematical problem solving: A path analysis. *Journal of Educational Psychology, 86*, 193-203.

Papadakis, S., Kalogiannakis, M., & Zaranis, N. (2017). Improving mathematics teaching in kindergarten with realistic mathematical education. *Early Childhood Education Journal, 45,* 369-378. doi:10.1007/s10643-015-0768-4

Perlmutter, J., Bloom, L., Rose, T., & Rogers, A. (1997). Who uses math? Primary children's perceptions of the uses of mathematics. *Journal of Research in Childhood Education, 12*, 58-70.

Plewis, I., Mooney, A., & Creeser, R. (1990). Time on educational activities at home and educational progress in infant school. *British Journal of Educational Psychology, 60,* 330–337. doi:10.1111/j.2044-8279.1990.tb00949.x

Purpura, D. J., & Reid, E. E. (2016). Mathematics and language: Individual and group differences in mathematical language skills in young children. *Early Childhood Research Quarterly, 36*, 259-268. doi:10.1016/j.ecresq.2015.12.020

Ramani, G. B & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development, 79*, 375-394. doi:10.1111/j.1467-8624.2007.01131

Ramani, G. B & Siegler, R. S. (2014). How informal learning activities can promote children's numerical knowledge. In R. C. Kadosh & A. Dowker (Eds.), *Oxford Handbook of Mathematical Cognition*. doi:10.1093/oxfordhb/9780199642342.013.012.

Rittle-Johnson, B. (2017). Developing mathematics knowledge. *Child Development Perspectives*, *11*, 184-190. doi:10.1111/cdep.12229

Rosenzweig, C., Krawec, J., & Montague, M. (2011). Metacognitive strategy use of eighth-grade students with and without learning disabilities during mathematical problem solving a think-aloud analysis. *Journal of Learning Disabilities*, *44*, 508-520. doi:10.1177/0022219410378445

Rothman, R. W., & Cohen, J. (1989). The language of math needs to be taught. *Academic Therapy, 25*, 133-142. doi:10.1177/105345128902500202

Rozek, C. S., Hyde, J. S., Svoboda, R. C., Hulleman, C. S., & Harackiewicz, J. M. (2015). Gender differences in the effects of a utility-value intervention to help parents motivate adolescents in mathematics and science. *Journal of Educational Psychology, 107*, 195-206. doi:10.1037a0036981

Rozek, C. S., Svoboda, R. C., Harackiewicz, J. M., Hulleman, C. S., & Hyde, J. S. (2017). Utility-value intervention with parents increases students' STEM preparation and career pursuit. *Proceedings of the National Academy of Sciences, 114*, 909-914. doi:10.1073/pnas.1607386114

Sarama, J. & Clements, D. H. (2006). 5 math myths. *Scholastic Parent & Child, 13*, 44-45.

Sarama, J. & Clements, D. H. (2007). How children problem solve. *Early Childhood Today, 21*, 16-19.

Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processes in early number development. *Monographs of the Society for Research in Child Development, 52,* 153–159. doi:10.2307/1166071

Schoenfeld, A. H. (1989). Explorations of students' mathematical beliefs and behavior. *Journal in Research in Mathematics Education, 20,* 338-355. doi:10.2307/749440

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics* (pp. 334-370). New York: Macmillan Publishing Company.

Seo, K.-H., & Ginsburg, H. P. (2004). What is developmentally appropriate in early childhood mathematics education? Lessons from new research. In D. H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 91–104). Hillsdale, NJ: Erlbaum.

Siegler, R. S., & Mu, Y. (2008). Chinese children excel on novel mathematics problems even before elementary school. *Psychological Science, 19*, 759-763. doi:10.1111/j.1467-9280.2008.02153.x

Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games—but not circular ones—improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology*, *101*, 545-560. 10.1037/a0014239

Simpkins, S. D., Fredricks, J. A., & Eccles, J. S. (2012). Charting the Eccles' expectancy-value model from mothers' beliefs in childhood to youths' activities in adolescence. *Developmental Psychology*, *48,* 1019-1032. doi:10.1037/a0027468

Singh, K., Granville, M., & Dika, S. (2002). Mathematics and science achievement: Effects of motivation, interest, and academic engagement. *The Journal of Educational Research*, *95*, 323-332. doi:10.1080/00220670209596607

Skwarchuk, S. L., Sowinski, C., & LeFevre, J. A. (2014). Formal and informal home learning activities in relation to children's early numeracy and literacy skills: The development of a home numeracy model. *Journal of Experimental Child Psychology*, *121*, 63-84. doi:10.1016/j.jecp.2013.11.006

Sonnenschein, S., Baker, L., & Garrett, A. (2004, December). Final Report of the 2004 Super Kids Camp. Sponsored by Parks and People Foundation.

Sonnenschein, S., Galindo, C., Metzger, S. R., Thompson, J. A., Huang, H. C., & Lewis, H. (2012). Parents' beliefs about children's math development and children's participation in math activities. *Child Development Research, 2012*, doi:10.1155/2012/851657

Sonnenschein, S., Metzger, S. R., & Thompson, J. A. (2016). Low-income parents' socialization of their preschoolers' early reading and math skills. *Research in Human Development, 13*, 207-224. doi: 10.1080/15427609.2016.1194707

Spinath, B., Spinath, F. M., Harlaar, N., & Plomin, R. (2006). Predicting school achievement from general cognitive ability, self-perceived ability, and intrinsic value. *Intelligence, 34*, 363-374. doi:10.1016/j.intell.2005.11.004

Thompson, R. J., Napoli, A. R., & Purpura, D. J. (2017). Age-related differences in the relation between the home numeracy environment and numeracy skills. *Infant and Child Development, 26*, 13 pages. doi:10.1002/icd.2019

Tsao, Y.-L. (2004). A comparison of American and Taiwanese students: Their math perceptions. *Journal of Instructional Psychology, 31*, 206-213.

Tudge, J., & Doucet, F. (2004). Early mathematical experiences: Observing young Black and White children's everyday activities. *Early Childhood Research Quarterly, 19*, 21–39. doi: 10.1016/j.ecresq.2004.01.007

U.S. Department of Education. (2013). *National Assessment of Education Progress.* National Center for Educational Statistics.

U.S. Department of Education. (2015a). *National Assessment of Education Progress.* National Center for Educational Statistics.

U.S. Department of Education. (2015b). *Trends in International Math and Science Study*. National Center for Educational Statistics.

U.S. Department of Education. (2018). *National Assessment of Education Progress.* National Center for Educational Statistics.

Van Oers, B. (2010). Emergent mathematical thinking in the context of play. *Educational Studies in Mathematics, 74*, 23-37. doi:10.1007/s10649-009-9225-x

Vo, V. A., Li, R., Kornell, N., Pouget, A., & Cantlon, J. F. (2014). Young children bet on their numerical skills metacognition in the numerical domain. *Psychological Science, 25*, 1712-1721. doi:10.1177/0956797614538458

Wang, Q. (2004). The emergence of cultural self-constructs: autobiographical memory and self-description in European American and Chinese children. *Developmental Psychology*, *40*, 3-15. doi:10.1037/0012-1649.40.1.3

Watts, T. W., Duncan, G. J., Siegler, R. S., & Davis-Kean, P. E. (2014). What's past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher*, *43*, 352-360. doi:10.3102/0013189X14553660

Wigfield, A. & Eccles, J. S. (2000). Expectancy-value theory of achievement motivation. *Contemporary Educational Psychology, 25*, 68-81. doi:10.1006/ceps.1999.1015

Wigfield, A., Eccles, J. S., Fredricks, J. A., Simpkins, S., Roeser, R. W., & Schiefele, U. (2015). Development of achievement motivation and engagement. In R. M. Lerner (Ed.) & W. F. Overton & P. C. M. Molenaar (Vol. Eds.), *Handbook of child psychology and developmental science: Vol. 3 Socioemotional processes* (7th ed., pp. 657-700). Hoboken, New Jersey: Wiley.

Wigfield, A., Eccles, J. S., Roeser, R. W., & Schiefele, U. (2008). Development of achievement motivation. In W. Damon, R. M. Lerner, D. Kuhn, R. S. Siegler, & N. Eisenberg (Eds.), *Child and adolescent development: An advanced course* (pp. 406-434). Hoboken, NJ: John Wiley & Sons, Inc.

Wigfield, A., Eccles, J. S., Schiefele, U., Roeser, R. W., & Davis-Kean, P. (2006). Development of achievement motivation. In W. Damon & R. M. Lerner (Series Eds.) & D. Kuhn & R.

Siegler (Vol. Eds.), *Handbook of child psychology: Vol. 3 Social, Emotional, and Personality Development* (6th ed., pp. 933-1002). New York: Wiley.

Wigfield, A., Eccles, J. S., Yoon, K. S., Harold, R. D, Arbreton, A. J. A., Freedman-Doan, C. & Blumenfeld, P. C. (1997). Change in children's competence beliefs and subjective task values across the elementary school years: A 3-year study. *Journal of Educational Psychology*, *89*, 451-469. doi:10.1037/0022-0663.89.3.451

Wigfield, A., & Guthrie, J. T. (1997). Relations of children's motivation for reading to the amount and breadth or their reading. *Journal of Educational Psychology*, *89*, 420-432. doi:10.1037/0022-0663.89.3.420

Wigfield, A. & Meece, J. (1988). Math anxiety in elementary and secondary school students. *Journal of Educational Psychology*, *80*, 210-216.

Woodcock, R. W., & Johnson, M. B. (1989). *Woodcock-Johnson PsychoEducational BatteryRevised. Tests of Achievement*, Itasca, IL: Riverside Publishing.

Table 1

Description of Measures

Constructs	Item	Codes or Scale
Knowledge about Math		
Math Concepts	"What is math?"	Content
		Number and operations *("math is numbers"*)
		Algebra (*"like when 7 minus X equals 2"*)
		Geometry (*"math is shapes"*)
		Measurement (*"we use rulers to see how many inches the eraser is"*)
		Data analysis and probability (*"when you make charts with everyone's eye color in the class"*)
		Processes
		Problem solving (*"math is solving problems"*)
		Reasoning and proof (*"logic is a part of math"*)
		Communication (*"when the teacher tells us to tell how we got our answer"*)
		Connections (*"math is in science and physics too"*)
		Representations (*"we use base 10 blocks to find the answers"*)
Applicability of Math	How do you learn math?	School (*"I learn at school," "from my teacher,"* or *"by doing your homework"*)
		Parents (*"with my daddy," "my parents help me with my homework,"* or *"playing math games on the computer with mommy"*)
		Learning math by doing it (*"you have to practice it"*)
		Other activities that are not specific to home or school ("*we learn math by using blocks"*)
		Friends/siblings (*"my sister helps me learn"*)
	How does {person} use math?	School-related uses (*"they teach students how to do math"*)
		Home-related uses (*"they play with math games at home"*)
		Math operations (*"they add and subtract things"*)
		Job-related math operations (*"she adds things when she's at work"*)
		Daily living math operations (*"he measures cups when he's cooking"*)

	Do you think that math is used when you do (activity)? How is math used in (activity)?	0 Child did not identify that math was used or if s/he said that math was used, but the description was not related to math (*"you read the instructions on the card"*). 1 Child said that math was used in the activity, but did not describe how (*"I don't know"*). 2 Child described a basic math skill, such as number recognition (*"when you play cards, there are numbers on them"*) or operation, such as counting (*"you count the spaces when you roll the die"*) 3 Child described an advanced math skill, such as number magnitude comparison (*"when you keep score, you have to know whose score is bigger so you know who won"*) or measurement (*"for the recipe, you have to measure ¼ cup of flour and 2 cups of sugar"*)
Beliefs about Math		
Utility Value	Math is useful outside of class. I need to learn math to do well in school. It is important for me to learn math. My parents think it is important for me to learn math. I think it is important for everyone to learn math. It is important for me to do well in math.	1 "Not at all like me" 2 "A little like me" 3 "A lot like me"
Productive Disposition	Who uses math?	Teachers Parents Children Other adults Everyone
Home Math Engagement		
	How often do you (activity) at home? (Example activities: play board games, play card games, help with cooking)	1 "Almost never" 2 "Sometimes" 3 "Almost every day"

Table 2

What is Math?

Coding Category	Overall *N* = 99	Rising 1st Grade *N* = 33	Rising 2nd Grade *N* = 23	Rising 3rd Grade *N* = 23	Rising 4th Grade *N* = 20
Content	91.9%	84.8%	91.3%	95.7%	100%
Number and Operations	89.9%	81.8%	91.3%	95.7%	95.0%
Counting	18.2%	24.2%	30.4%	4.3%	10.0%
Number Knowledge	2.0%	0.0%	4.3%	4.3%	0.0%
Number Transformations	66.7%	45.5%	65.2%	82.6%	85.0%
Number Patterns	6.1%	17.4%	30.0%	0.0%	0.0%
Algebra	3.0%	0.0%	0.0%	4.3%	10.0%
Geometry	8.1%	12.1%	8.7%	0.0%	10.0%
Measurement	7.1%	6.1%	8.7%	4.3%	10.0%
Processes	12.1%	6.1%	4.3%	21.7%	20.0%
Problem Solving	5.1%	0.0%	4.3%	8.7%	10.0%
Connections	5.1%	3.0%	0.0%	8.7%	10.0%

Note. Children were able to give more than one response; therefore percentages do not total 100%.

Table 3

How Do You Learn Math?

	Overall *N* = 99	Rising 1st Grade *N* = 33	Rising 2nd Grade *N* = 23	Rising 3rd Grade *N* = 23	Rising 4th Grade *N* = 20
School	73.7%	51.5%	65.2%	100.0%	90.0%
Teachers	54.5%	39.4%	52.2%	65.2%	70.0%
Homework	5.1%	3.0%	0.0%	8.7%	10.0%
Parents	27.3%	24.2%	17.4%	39.1%	30.0%
School Work	1.0%	0.0%	0.0%	4.3%	0.0%
Home Activities	2.0%	0.0%	0.0%	8.7%	0.0%
By doing math	8.1%	6.1%	4.3%	13.0%	10.0%
Activities (not home/school)	12.1%	18.2%	17.4%	4.3%	5.0%
Friends/Siblings	5.1%	0.0%	0.0%	8.7%	15.0%

Note. Children were able to give more than one response; therefore percentages do not total 100%.

Table 4

How Does {Person Mentioned} Use Math? By Grade and Uses

	Overall *N* = 99	Rising 1st Grade *N* = 33	Rising 2nd Grade *N* = 23	Rising 3rdGrade *N* = 23	Rising 4th Grade *N* = 20
School	35.4%	32.1%	46.3%	33.3%	30.3%
Home	0.6%	0.0%	2.4%	0.0%	0.0%
Math Operations	61.8%	53.6%	46.3%	72.9%	78.8%
Job-related	24.2%	8.9%	14.6%	41.7%	36.4%
Daily Living	5.1%	0.0%	2.4%	10.4%	9.1%

Note. Children were able to give more than one response; therefore percentages do not total 100%.

Table 5

How Does {Person Mentioned} Use Math? By Person Mentioned and Uses

	Teachers	Parents	Children	Other Adults	Everyone
School	76.2%	13.3%	39.3%	6.5%	14.3%
Home	0.0%	0.0%	1.6%	0.0%	0.0%
Math Activities	33.3%	60.0%	54.1%	89.1%	92.9%
Job-related	9.5%	33.3%	0.0%	69.6%	14.3%
Daily Living	0.0%	13.3%	4.9%	2.2%	21.4%

Note. Children were able to give more than one response; therefore percentages do not total 100%.

Table 6

Who Uses Math? Responses by Grade

Coding Category	Overall *N* = 99	Rising 1st Grade *N* = 33	Rising 2nd Grade *N* = 23	Rising 3rd Grade *N* = 23	Rising 4th Grade *N* = 20
Teachers	43.4%	42.4%	69.6%	34.8%	25.0%
Parents	18.2%	21.2%	8.7%	26.1%	15.0%
Children	52.5%	63.6%	56.5%	43.5%	40.0%
Other Adults[a]	38.4%	18.2%	34.8%	60.9%	50.0%
Everyone	16.2%	9.1%	8.7%	21.7%	30.0%

Note. Children were able to give more than one response, so percentages do not total 100%.
[a]Other adults include scientists, architects, accountants, adult relatives, cashiers, engineers, and mathematicians.

Table 7

Correlations Between Children's Math Applicability Scores for Various Activities, Controlling for Grade and Length of Utterance

	1	2	3	4	5	6	7	8	9
(1) Board Games	-	-	-	-	-	-	-	-	-
(2) Cooking	.11	-	-	-	-	-	-	-	-
(3) Grocery Store	.03	.16	-	-	-	-	-	-	-
(4) Keep Score	.28*	.33**	.02	-	-	-	-	-	-
(5) Playing Cards	-.04	.20†	.10	-.05	-	-	-	-	-
(6) Blocks/ Legos	.14	.24*	.05	.09	.35***	-	-	-	-
(7) Video Games	.15	.09	.09	.15	.06	.05	-	-	-
(8) Money	.15	.29*	.06	.32**	.19†	.10	-.07	-	-
(9) Puzzles	.14	.06	.17	.13	-.04	.17†	.14	-.06	-
(10) Maps	.01	.16	.21†	.11	<.01	.14	<.01	.20†	.17

Note. $^{\dagger}p \leq .10$, $^{*}p < .05$, $^{**}p < .01$, $^{***}p \leq .001$

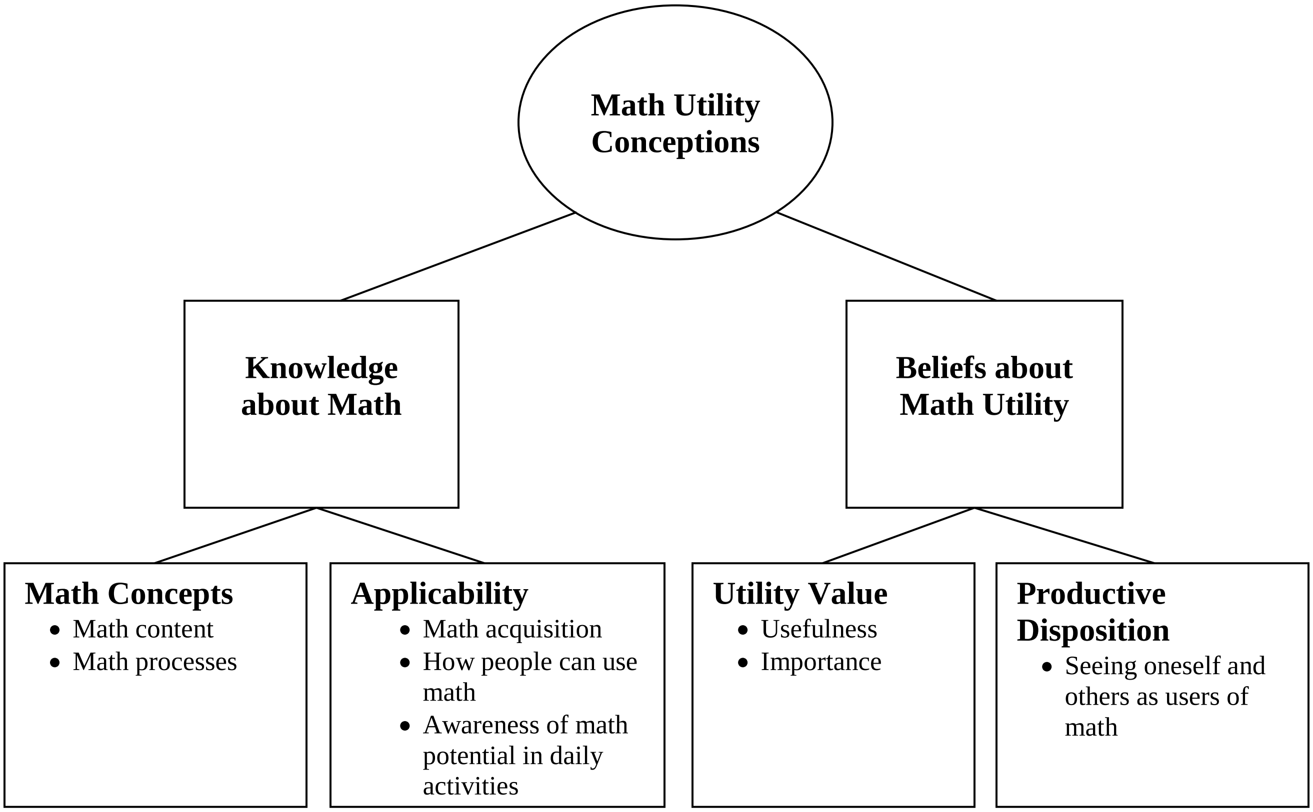

Figure 1. Math utility conceptions model.

Chapter 3

Elementary-School Age Children’s Math Utility Conceptions, their Home-Based Math Engagement, and their Parents’ Math Utility Conceptions

Children in the United States have a long-documented history of low achievement on national and international standardized math tests (National Mathematics Advisory Panel, 2008; U.S. Department of Education, 2014, 2015a, 2015b, 2018). To increase the number of children demonstrating proficiency in mathematics, it is important to determine the factors that contribute to mathematical learning. One factor that is associated with such learning is children’s math conceptions (Muis, 2004; National Research Council, 2001), which include children’s knowledge, attitudes, and beliefs about math. The National Research Council (2001) posits that children’s beliefs that math is useful and worthwhile, called productive disposition, is an important component for children to develop math proficiency.

Research on children’s math utility conceptions shows that these conceptions are positively associated with children’s math development (Mazzocco, Hanich, & Noeder, 2012; Rozek, Hyde, Svoboda, Hulleman, & Harackiewicz, 2015). However, these few studies only considered limited components of math utility conceptions, typically children’s utility beliefs, and have not considered children’s knowledge about how math is useful in daily life (Eccles, Wigfield, Harold, & Blumenfeld, 1993; Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002; National Research Council, 2001). Thus, we know relatively little about children’s understanding of what math is and how they think it can be applied in their daily lives. To better understand children’s math utility conceptions, it is important to examine two key constructs: children’s *knowledge about math,* including math concepts and applicability of math, and *beliefs about* *math utility*, including utility value and productive disposition (see Figure 1 for a visual representation of the constructs measured in the present study).

In addition to studying children's math utility conceptions, it is important to understand how their environment may impact those conceptions. Research shows that young children's numeracy environments impact their early math skills (LeFevre et al., 2009; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010). However, less is known about how children's environments impact their development of math conceptions. Results from Metzger, Sonnenschein, and Galindo (2018, Study 1) provided insight into children's conceptions of math utility, but not how these conceptions develop. They found that frequency of children's engagement in math-related activities at home did not relate to children's awareness of math in their daily activities. This may have been due partially to the extent to which children engaged in the math-related aspects of home activities. This suggests that other aspects of children's home numeracy environments, such as parents' conceptions, may impact their math utility conceptions. Parents' beliefs about children's math activities are associated with children's math achievement (Saxe, Guberman, & Gearhart, 1987; Sonnenschein, Metzger, & Thompson, 2016). Parents' math utility conceptions, which include their beliefs about the usefulness of math, may also be related to children's math utility conceptions. This study examines potential home influences on children's conceptions about the utility of math and the relations between parent conceptions, children's conceptions, and children's home engagement in math activities.

Children's Math Conceptions

The conceptual model in this study includes knowledge and beliefs about math utility. Knowledge about math concepts includes content and processes as defined by the National Council of Teachers of Mathematics (NCTM, 2000). Content includes number and operations, algebra, geometry, measurement, and data analysis and probability. Processes include problem solving, reasoning and proof, communication, connections, and representations. Children must develop a sense of what math is before they can develop notions about its usefulness, and children's ability to assess their own knowledge of math is related to their math learning and achievement (Dunlosky & Rawson, 2012; Vo, Li, Kornell, Pouget, & Cantlon, 2014). Knowledge of both math content and math processes are important for the development of math skills (National Research Council, 2001; Rittle-Johnson, 2017).

Beliefs about math utility include utility value beliefs and the extent to which children have a productive disposition, or attitude toward math (National Research Council, 2001). Math utility value is positively related to math achievement and high school math course selection (Guo, Marsh, Parker, Morin, & Yeung, 2015; Marsh & Martin, 2011; Singh, Granville, & Dika, 2002). The National Research Council (2001) recognized the importance of math utility by including productive disposition, children's beliefs that they are users of math, and that math is useful and worthwhile, as one of their five "strands" of math proficiency. The current study expands on Eccles' math utility value research (Eccles & Wigfield, 2002; Eccles et al., 1993) and the limited research on productive disposition.

Developmental changes in children's math utility conceptions. As children progress through school, they build on their knowledge and understanding within each math content area (Clements & Sarama, 2014; Geary, 2006; Rittle-Johnson, 2017). Knowledge about math applicability refers to children's knowledge that math is used in their daily lives. Young children, when exposed to continued "real-world" math in their environment can learn more about how math is used in daily activities (Clements & Sarama, 2007; Papadakis, Kalogiannakis, & Zaranis, 2017; Permutter, Bloom, Rose, & Rogers, 1997). Little research has examined grade level differences in children's definitions of what math is and whether they believe it is learned or used outside of the school context.

Several studies have shown that children's expectancy-value beliefs in math decline from elementary school through high school (Eccles et al, 1993; King & McInerney, 2014; Muenks, Wigfield, & Eccles, 2018; Musu-Gilette, Wigfield, Harring, & Eccles, 2015; Nagy et al., 2010), but little research has examined developmental differences in these beliefs in early elementary school. For example, Musu-Gillette et al. (2015) found that, on average, utility value was highest in fourth grade, and most children showed an overall decline through early college, although rates of decline differed. Some research suggests that math utility beliefs can improve through interventions (Hulleman, Kosovich, Barron, & Daniel, 2017; Jansen, 2012; Mitchell, 1999). This study builds on prior research by examining grade-level differences in knowledge and beliefs about math utility for early elementary-age children.

Parents' Math Conceptions

Research generally shows that parents' math practices at home and beliefs about math are associated with preschool and kindergarten children's early math skills (e.g., Blevins-Knabe & Musin-Miller, 1996; Ramani, Rowe, Eason, & Leech, 2015; Sonnenschein et al., 2016). Parents' early math talk relates to preschool and kindergarten children's number knowledge (Blevins-Knabe & Musin-Miller, 1996; Gunderson & Levine, 2011; LeFevre et al., 2009; Levine et al., 2010). Additionally, the extent to which parents of preschool to first grade children like math and the frequency with which their children see them engage in math-related activities significantly predicted children's engagement in math activities at home, which, in turn, significantly predicted children's early math skills (Sonnenschein et al., 2016). However, the association between parent beliefs and children's beliefs, a component of math utility conceptions, is still largely understudied.

Early research into parents' beliefs about math focused primarily on parents' beliefs about their role in preparing their children for school (Barbarin et al., 2008; Okagaki & Sternberg, 1993) and, relatedly, the numeracy environment they create in their homes (Saxe et al., 1987). Although knowledge about the home numeracy environment is growing, fairly little is known about parents' beliefs about math, how parents view their role in teaching math to their children, and the full extent of the activities they make available to their children (Huntsinger, Jose, Larson, Balsink Krieg, & Shaligram, 2000). Sonnenschein and colleagues (Sonnenschein et al., 2012, 2016, 2018) have conducted more in-depth research on parents' beliefs about math and how they socialize math with their young children. They found that parents of preschool to first grade children reported that they believe it is important for their children to do math at home and for parents to help their children with math. Additionally, they reported that their children engaged daily in at least one math activity.

It is important to understand parents' beliefs about math, because parent beliefs are often associated with their parenting behaviors (Missall, Hojinski, Caskie, & Repasky, 2015; Okagaki & Bingham, 2005; Schofield & Weaver, 2015). If parents believe it is their role to engage their children in math activities at home, they may have their children more frequently engage in math-related games/activities (e.g., building with blocks, measuring ingredients for cooking, and playing various counting games) that have the potential to improve children's math knowledge (Clements & Sarama, 2006, 2014; Ginsburg, Lee, & Boyd, 2008; Sarama & Clements, 2006). Parents also serve as role models for math engagement. For example, the frequency with which preschool through first grade children see their parents using math (paying bills, cooking, or counting money at the grocery store, etc.) is positively associated with their home math engagement (Sonnenschein et al., 2016). This link between parents' beliefs about math and the numeracy environments they create for their children highlights the importance of understanding how parents' math utility conceptions may be associated with children's conceptions.

Eccles and colleagues (Eccles, 2009; Eccles et al., 1993; Muenks et al., 2018; Parsons [Eccles], Adler, & Kaczala, 1982; Wigfield & Eccles, 2000) have studied children's math utility-value, how it is acquired, and how it relates to their achievement and choice of advanced math classes in high school. Their theoretical model posits that is important for parents to serve as positive role models for math (Simpkins, Fredricks, & Eccles, 2012), but few studies have examined how parents' math utility value is associated with their children's math utility values. Harackiewicz, Rozek, Hulleman, and Hyde (2012) explored whether informing parents of high school children about the usefulness of advanced math and science courses in high school would impact their course selections. They used a simple intervention to encourage parents to discuss the utility value of taking advanced math and science courses with their high school children. Children whose parents discussed the utility value of math and science courses for their lives and future careers took an average of one additional math or science class in their last two years of high school. Additional research has found that both parents' beliefs about their high school children's academic abilities and the extent to which parents value academics positively predicts children's academic utility value (Chouinard, Karsenti, & Roy, 2007; Gniewosz & Noack, 2012; Jodl, Michael, Malanchuk, Eccles, & Sameroff, 2001; Lazarides, Harackiewicz, Pesu, & Viljaranta, 2015; Simpkins, Fredricks, & Eccles, 2015; Viljaranta, Lazarides, Aunola, Räikkönen, & Nurmi, 2015). For example, mothers' beliefs that math is a useful subject positively predicts their children's beliefs about the usefulness of math (Harackiewicz et al., 2012). It is possible that if parents engage elementary school children in discussions about math utility, this may impact their children's conceptions about math utility in a similar way.

Although parents' beliefs about math predict children's beliefs, some research suggests that this effect may be moderated by other factors in the home environment (Maloney, Ramirez, Gunderson, Levine, & Beilock, 2015). For example, Maloney et al., 2015 found that the extent to which parents' beliefs about math predict young children's beliefs and achievement is moderated by how often parents spend time helping their children with homework. More specifically, when parents helped their children more frequently with math homework, parents' math anxiety is positively associated with first and second grade children's math anxiety and negatively associated with children's math achievement (Maloney et al., 2015). If they helped their children less often with math homework, parents' math anxiety was not related to children's. The present study examines the association between parents' and children's conceptions about math utility as well as explores the potential moderating effects of children's home numeracy environment, including how often parents help their children with homework, how frequently their children are engaged in math activities at home, and the extent to which their children see their parents using math.

Children's Home-Based Math Engagement

Very young children gain mathematics knowledge from their environment before they start school (Ginsburg et al., 2008; LeFevre et al., 2009; Sarama & Clements, 2006, 2007; Siegler & Mu, 2008). Children develop "everyday mathematics" concepts, including size, shape, and magnitude by engaging in activities such as building with blocks, measuring ingredients for cooking, and playing various counting games (Clements & Sarama, 2007; Ginsburg et al., 2008). Engaging in everyday math activities at home positively predicts the development of early math skills (Blevins-Knabe & Musun-Miller, 1996; Clements & Sarama, 2006; Downer & Pianta, 2006; LeFevre et al., 2009; LeFevre, Polyzoi, Skwarchuk, Fast, & Sowinski, 2010; Skwarchuk & LeFevre, 2015; Sonnenschein et al., 2016). Ramani and Siegler (2008) found that the frequency with which preschool children reported engaging in board games—especially those using dice or spinners, card games, and video games outside of school predicted aspects of their number knowledge at the beginning of preschool. Similarly, LeFevre and colleagues (LeFevre et al., 2009, 2010) found that the frequency with which young children engaged in home-based math activities such as playing board games, card games, cooking, and shopping predicted their math knowledge and fluency.

The limited research on the association between home engagement and children's math conceptions suggests that children begin to develop such conceptions, in particular, self-efficacy and self-concepts beliefs, through home experiences (for a review, see Muis, 2004). This suggests that children's conceptions of math utility may be related to their math engagement at home. However, the relation between home math engagement and math utility conceptions have not been well-documented. Existing research focuses primarily on early math skills as the outcome rather than children's math conceptions (e.g., Laski & Siegler, 2014; Ramani & Siegler, 2008; Skwarchuk & LeFevre, 2015). Consequently, little is known about the relations between math home engagement and math utility conceptions.

Most of what is known about children's engagement in math activities at home is parent-reported frequency of young (preschool and early elementary) children's engagement in such activities (e.g., Saxe et al., 1987; Skwarchuk & LeFevre, 2015; Sonnenschein et al., 2016). For example, Saxe et al. (1987) conducted in-depth interviews with mothers of preschool age children to examine the frequency of engagement at home for many math-related activities, including store-bought games, number books, and games that the parents created that involve numbers or counting. Sonnenschein and colleagues (Sonnenschein et al., 2012, 2016) measured parent-reported frequency of preschool through first grade children's math activities, including counting, writing numbers, discussing quantities, playing math board games, and watching math television programs. While this provides information about which activities very young children are engaged in at home, engagement in math activities at home for older children (early to middle elementary school age) is relatively unknown.

Present Study

The goals of this study are to examine children's math utility conceptions, developmental differences in those conceptions, and how the home numeracy environment, specifically parents' math utility conceptions and children's math engagement at home, relate to those conceptions. Research suggests that parents' beliefs and modeling behaviors are associated with children's skills (Sonnenschein et al., 2016). Similarly, it is expected that parents' math utility conception scores will positively predict children's math utility conception scores.

Furthermore, Maloney et al. (2015) found that parents' math anxiety positively predicted children's math anxiety, but only when parents spent more time helping their children with math homework. If parents' math anxiety is more associated with children's anxiety when they spend more time engaging with their children in a math-related activity, there may be similar moderators of the relation between parents' and children's math utility conceptions. We explore the potential moderating effects of three aspects of the home numeracy environment: the frequency with which parents help their children with math homework, children engage in math activities at home, and parents model their own math engagement. Based on Maloney et al.'s (2015) results, we hypothesize that the extent to which parents' math utility conceptions predict children's conceptions will vary based on these three variables. Specifically, when each of these aspects of the home numeracy environment is higher, the association between parents' and children's math utility conceptions will be stronger.

This study adds to the current literature by expanding on current measures of math utility conceptions by measuring both knowledge and beliefs about children's and parents' math utility conceptions. Additionally, the present study explores the potential moderating effects of the home numeracy environment on the relation between parents' math utility conceptions and children's conceptions. Finally, this study adds to current research related to the association between children's home math engagement and their math utility conceptions.

Method

Participants

Children. One hundred and four children (55% girls) were recruited during the summer and early fall from schools and summer camps in the Baltimore/Washington, D.C. corridor. Most of the children (88%) were interviewed during the summer. Twenty-eight children (Mean age = 6.60 years, *SD* = 0.41, 43% girls) were entering or had just entered first grade, 26 second grade (Mean age = 7.36 years, *SD* = 0.31, 58% girls), 26 third grade (Mean age = 8.29 years, *SD* = 0.35, 62% girls), and 24 fourth grade (Mean age = 9.41 years, *SD* = 0.36, 58% girls) in the fall following their interview. Consistent with similarities in math instruction and some previous research with this age group (Simons, Metzger, & Sonnenschein, 2018; Sobel & Letourneau, 2015), children were grouped into younger (rising first and second graders, *n* = 54) and older (third and fourth, *n* = 50) grade groups.

Children came from diverse racial/ethnic backgrounds. Fifty-seven percent of child participants were European American/White (*n* = 59), 15% African American/Black (*n* = 16), 14% Asian/Pacific Islander (*n* = 14), 3% Hispanic/Latino (*n* = 3), 3% another race/ethnicity (*n* = 3), and 9% multiracial (*n* = 9). Slightly more than half of child participants had parents who were highly educated [57% (*n* = 59) earned a post-graduate degree, 28% (*n* = 29) Bachelor's degree, and 15% (*n* = 16) some college or an Associate's degree]. Additionally, 54% (*n* = 56) of children had parents who reported a household income of $125,000 or more, 32% (*n* = 33) $75,000 - $124,999, and 10% (*n* = 10) less than 75,000.

Parents. Ninety-eight percent (*n* = 102) of child participants also had a parent or primary caregiver participant in the study. However, because there were four sibling pairs in this study, there were only 94 (83% female) unique parent participants (81% (*n* = 76) mothers, 15% (*n* = 14) fathers, 2% (*n* = 2) grandfathers, and 2% (*n* = 2) other relatives). Mean parent age was 41.42 years (*SD* = 6.65). Participants came from a diverse racial/ethnic background: European American/White (*n* = 63, 67%), African American/Black (*n* = 12, 13%), Asian/Pacific Islander (*n* = 14, 15%), Hispanic/Latino (*n* = 2, 2%), another race/ethnicity (*n* = 2, 2%), or multiracial (*n* = 1, 1%).

Measures

Complete versions of the *Mathematics Conceptions Questionnaires* for children and parents can be found in Appendices A and B. Below is a description of specific measures within the questionnaires for each component of the math utility conceptions model and children's home math engagement.

Knowledge about math.

Math concepts. Both children and parents were asked "What is math?" To measure knowledge of math content and processes, responses were coded categorically based on NCTM's (2000) content (numbers and operation, algebra, geometry, data analysis and probability, etc.)

and process (problem solving, connections, etc.) standards. A detailed description of codes and exemplary quotes can be found in Table 1. For this and other open-ended responses, inter-rater reliability was established by having two raters independently code about 15-30% of the responses for each item. The researchers met after coding the transcripts to review their codes and reached consensus. Inter-coder reliability was tested using Cohen's kappa (Cohen, 1960). A kappa guideline of .70 was used to determine acceptable inter-rater reliability (Fleiss, 1981; Landis & Koch, 1977). If acceptable kappa levels were not reached in the first round of coding, the coding scheme was reviewed and modified, if necessary, and a new set of responses were coded. This process continued until kappas were at least .70 for every coding category. Discrepancies were resolved for responses coded by both raters, and remaining responses were coded by one of the raters who had reached acceptable reliability. Final kappas for math concepts were 1.00 for all codes.

To measure the extent to which children and parents define math as a useful tool, a scale based on Mazzocco et al. (2012) was developed (see Table 1). This five-point scale ranged from a 0, ("*I cannot explain"*) to 2 (*"It has to do with numbers"*) to 4 ("*Math is an important thing you use every day, like at the grocery store"*). This scale score represents the *math concepts* score within the overall math utility composite.

Inter-rater reliability for this and other scales was established by having two raters independently code about 15-30% of the responses. Because the data are on a continuous scale, an intraclass correlation coefficient (ICC, McGraw & Wong, 1996) was used to determine acceptable inter-rater reliability. Specifically, two-way random-effects model ICCs with absolute agreement (Koo & Li, 2016) were used. If acceptable ICC values were not reached in the first round of coding, a new set of responses were coded. This process continued until ICC values were at least .75, representing good reliability (Koo & Li, 2016). Remaining responses were coded by one of the raters who had reached acceptable reliability. The final ICC for the math concepts scale was .84.

Applicability of math.

Children. Children were asked "How do you learn math?" Descriptive codes represent the setting and/or from whom they believe math is learned and/or how they acquire math knowledge.

Children were asked, "Who uses math?" and "How does {person[s] mentioned} use math?" Similar to math concepts, a scale was developed based on children's responses to these questions to represent the extent of children's knowledge about how math is useful in daily life (see Table 1). The five-point scale ranged from a 0, ("*I cannot think of anyone"*) to 2 (*"everyone because they have to learn it in school"*) to 4 (*"Everyone has to use math in their life all the time- like at the grocery store"*). This scale score represents a *math uses* score within the math applicability composite. The final ICC for the math uses scale was .80.

To measure awareness of math in daily activities, children were asked whether and how mathematics can be used in 10 different activities: playing board games, card games, and video games, doing puzzles, cooking, helping at the grocery store, building with blocks or Legos, using or playing with money, using maps or a globe, and keeping score in games or sports. Children were asked, "Some children think math is used when they play board games, some think math is not used at all. Do you think math is used when you play board games?" If children responded "yes," they were asked how math was used in the activity. As in Metzger et al. (2018, Study 1), children's responses were coded on a 4-point scale (see Table 1): 0 if the child did not identify that math was used or if s/he said that math was used, but the description was not related to math;

1 if the child said that math was used in the activity, but did not elaborate about how; 2 if s/he described a basic math skill; 3 if s/he described an advanced math skill. The final ICCs for children's math awareness items ranged from .76 to .98. A composite was created by averaging scores for all activities. This composite scale score represents the *math awareness* score within the math applicability composite. Cronbach's alpha for the math awareness scale for children was .82 (.81 for younger children, .76 for older children). A composite was created by summing the *math uses* and *math awareness* scale scores. This scale score represents the *math applicability* score within the overall math utility composite.

Parents. Parents were asked whether they whether and how math featured in 10 activities or locations: decorating a home, in the kitchen, gardening/mowing the lawn, using a cell phone, planning a party or get-together, making art, travelling, playing or watching sports, making/listening to music, and at a restaurant. Parents were asked "Is math used in the kitchen?" If they responded "yes," they were asked how math was used in the activity or place. A scale similar to the children's scale was developed based on parents' responses (see Table 1). The final ICCs for parents' items ranged from .79 to 1.00. A composite was created by averaging scores for all activities/locations. This scale score represents the *math applicability* score within the overall math utility composite. Cronbach's alpha for the math applicability scale for parents was .82.

Beliefs about math.

Utility value. Children and parents were asked to rate nine items on a five-point scale to measure their utility value for math (see Table 1). Items were adapted from similar measures used to measure mathematics or reading motivation (Baker & Scher, 2002; Eccles et al., 1993; Sonnenschein, Baker, & Garrett, 2004; Wigfield & Guthrie, 1997). Examples included "Math is useful for everyone" and "It is important for me to learn math." Children were asked to report whether they felt each item is 1 "not at all like me," to 5 "a lot like me." Three non-math activities were presented as examples at the outset to familiarize children with the rating scale. Parents were asked to indicate the extent to which they agreed or disagreed with the same statements as the children (1 "strongly disagree" to 5 "strongly agree"). A composite was created by averaging the scores on the nine items. This composite scale will represent *utility value* score within the overall math utility composite. Cronbach's alpha for the utility value scale for children was .79 (.79 for younger children, .80 for older children) and for parents was .78.

Productive disposition. Children and parents were asked to rate several items (10 for children, nine for parents, see Table 1) on a five-point scale to measure their disposition towards math. Some items were adapted from similar measures used to measure mathematics or reading motivation (Baker & Scher, 2002; Eccles et al., 1993; Sonnenschein et al., 2004; Wigfield & Guthrie, 1997). Examples included "Working hard in math helps me do better in math" and "Learning math is worth my time." Children were asked to report whether they felt each item is 1 "not at all like me," to 5 "a lot like me." Parents were asked to indicate the extent to which they agreed or disagreed (1 "strongly disagree" to 5 "strongly agree") with the same statements, except for "I know that I will do well in math this year," which parents were not asked. A composite was created by averaging the scores on all items. This composite scale represented *productive disposition* score within the overall math utility composite. Cronbach's alpha for the productive disposition scale for children was .87 (.87 for younger children, .88 for older children) and for parents was .91.

Overall math utility conceptions. The scale scores for each component of the math utility conceptions model (math concepts, math applicability, math utility, and productive disposition) were standardized within group (children/parents), then summed.

Children's home math engagement. Both children and parents were asked independently to report children's math engagement at home by responding to how frequently children engaged in 14 activities: board games, math games on the computer, games or sports where someone keeps score, card games, video games, help with cooking, help at the grocery store, watch math TV shows, using money, math workbooks or flashcards, blocks or Legos, maps or globes, using a calculator, and puzzles. Response choices on the 5-point scale ranged from 1 "never/almost never" to 5 "every day/almost every day." Separate composite scores were created for parents and children by averaging the scores on all items. The composite score represented overall *home math engagement*. Cronbach's alpha for the child-reported home math engagement scale was .70 and the parent-reported home math engagement was .71. Child- and parent-reported home math engagement composites were significantly correlated, $r(99) = .33$, $p = .001$.

Demographic information. As part of the consent documents, parents provided their child's age, grade in school in the fall, gender, and race/ethnicity (African American/Black, European American/White, Hispanic/Latino, Asian/Pacific Islander, or "other"). Additionally, parents provided their age, race/ethnicity, relation to the child (mother, father, or "other"), highest level of education they earned (post-graduate degree, Bachelor's degree, Associate's degree, some college/vocational/technical, high school graduate, or less than high school), and total household income ($125,000 or more, $100,000 - $124,999, $75,000 - $99,999, $50,000 - $74,999, $25,000 – $49,999, or less than $25,000).

Length of Utterance

Several analyses in this study examined grade group differences in children's conceptions using responses to open-ended questions. This mixed-methods approach results in individual differences in sophistication and length of responses and may be biased towards older children (Sobel & Letourneau, 2015). Due to developmental differences in children's verbal skills between early and later elementary school grades (e.g., Hoff, 2014), we attempted to control for potential developmental differences in two ways. First, the coding schemes described above were designed to control for developmental differences in children's language sophistication. For example, in response to "How do you use math when you cook?" a child who said "you measure how many cups you need" and a child who gave a detailed account of measuring and used several measurement terms received the same score. Also, we completed a length of utterance analysis for each open-ended item to control for the potential developmental differences in the length of children's responses as well as the potential for longer responses to mention more coding categories. All words other than filler words such as "um" and "uh" were counted.

Procedure

Children were interviewed individually by a trained graduate student or advanced undergraduate research assistant. Each interview took place in the investigator's research lab, the child's home, or a public library and lasted approximately 15-20 minutes. Researchers administered the *Children's Mathematics Conceptions Questionnaire* and responses were recorded using a digital recorder. The interviewer also took notes of the child's responses. Parents completed the *Parents' Math Conceptions Questionnaire* when they filled out the participant consent forms. All interviews and questionnaires were in English.

Results

Preliminary Analyses

Preliminary analyses were conducted to determine which variables, if any, should be included as covariates in analyses. First, we examined whether length of utterance needed to be used in analyses. Overall, independent-samples t-tests did not show significant length of utterance differences between grade groups for most open-ended item responses. However, older children had significantly higher sum of length of utterances for the questions that featured into the math utility conceptions scale score, $t(102) = 2.20$, $p = .030$ and overall length of utterance for the entire interview, $t(102) = 1.98$, $p = .050$, than younger children. Additionally, after controlling for grade, the length of utterances for all open-ended items in the overall math utility conceptions composite, $r(99) = .25$, $p = .012$, and the total length of utterance throughout the entire interview, $r(99) = .29$, $p = .003$, were both significantly correlated with children's overall math utility conceptions score. Accordingly, any analyses predicting children's math utility included the appropriate length of utterance as a covariate.

Next, independent-samples t-tests were used to determine whether there were gender differences in math utility conceptions. There were no significant gender differences on any of the math utility composite scores, the home math engagement composite score, any of the individual math utility or productive disposition individual items, or on any of the coding categories across all open-ended items, $p > .05$. The only gender differences were that boys reported playing games or sports where they keep score, $p = .021$, video games, $p < .001$, and with blocks or Legos more frequently than girls, $p < .001$. Because of the overall lack of significant gender differences, gender was not used as a covariate in analyses.

Finally, we examined whether socioeconomic factors, parents' highest education earned and household income, were correlated with children's math utility conceptions. Parents' highest education level was significantly correlated with children's math utility scale scores, $r(102) = -.28$, $p = .004$, and children's productive disposition scale score, $r(102) = -.19$, $p = .052$, but not significantly correlated with math knowledge scores, $p = .197$, math applicability scores, $p = .695$, the overall math utility conceptions composite, $p = .071$, or home math engagement, $p = .778$. Household income was not significantly correlated with children's math knowledge scale scores, $p = .886$, math applicability, $p = .083$, math utility scale scores, $p = .516$, productive disposition scale score, $p = .223$, overall math utility conceptions, $p = .209$, or home math engagement, $p = .089$. Because socioeconomic factors were not significantly correlated with the outcomes in this study, overall math utility conceptions and home math engagement, neither were included in analyses.

Children's Math Utility Conceptions

Math concepts. Consistent with results of Metzger et al. (2018, Study 1), when asked "What is Math?" the majority of children (89%) defined math as number and operations, particularly number transformations (73%, see Table 2). Only 20% of children mentioned math processes. Eleven percent of children defined math as something that is fun (positive affect) and 8% of children defined math as something that increases your intelligence in a general way (math makes you smarter).

Chi-square tests were used to determine whether there were differences in children's math concepts knowledge between younger and older children. Significantly more older children (82%) than younger children (65%) mentioned number transformations (or calculations), $\chi^2(1, N = 104) = 3.90$, $p = .048$, $\Phi = .19$. Also, significantly more older children (22%) than younger children (4%) mentioned connections, $\chi^2(1, N = 104) = 7.95$, $p = .005$, $\Phi = .28$. There were no other significant differences. These differences suggest that older children may be defining math as the operations they are learning in school, but also may be acquiring knowledge about the interdisciplinary and applicable nature of math.

The mean math concepts scale score was 2.11 (*SD* = 0.75) on a 0-4 scale. Grade group differences in math concepts, math applicability, math utility, productive disposition, and overall math conceptions scale scores were tested using an OLS regression model with grade group as the predictor and the appropriate length of utterance as a covariate. Older children (*M* = 2.32, *SD* = 0.87) had significantly higher math concepts scale scores than younger children (*M* = 1.91, *SD* = 0.56), *B* = 0.39, $t(101) = 2.86$, $p = .005$, partial $\eta^2 = .075$. Most children (78% overall, 85% first/second graders, 70% third/fourth graders) received a score of two on the math concepts scale, which reflects knowledge of math focused primarily on numbers and operations. Few (14% overall, 4% first/second graders, 26% third/fourth graders) provided responses that indicated that they defined math as something that is connected to other subjects or daily life activities (three or four on the scale).

Math applicability. Most children mentioned learning math from school (88%) and teachers (71%). A little more than one-third of children mentioned learning math at home (39%) and from parents (34%, see Table 3). Chi-square tests were used to determine whether there were grade group differences. There were no significant grade group differences in children's responses, suggesting that children, regardless of grade group, view math as learned primarily in school.

When asked to identify users of math, children most often mentioned children (46%), including themselves and/or their peers/classmates/siblings, or that everyone uses math (45%, see Table 4). Thirty-nine percent of children mentioned other adults that use math, and the most common adults they mentioned were scientists, adult non-parent relatives, mathematicians, engineers, and carpenters/builders/architects. Less than one-third of children mentioned teachers (31%), and even fewer mentioned parents (15%). Chi-square tests were used to determine whether there were differences between younger and older children with whom they see as math users. Significantly more younger children (56%) than older children (36%) mentioned children, $\chi^2(1, N = 104) = 4.00$, $p = .046$, $\Phi = .20$. There were no other significant differences in other coding categories.

The mean math uses scale score (based on responses about who uses math and how) was 2.28 (*SD* = 1.24) on a 0-4 scale. Although 40% (56% first/second graders, 24% third/fourth graders) of the sample received a one on the math uses scale, 26% (17% first/second graders, 36% third/fourth graders) received a four, demonstrating that about one-fourth of the sample indicated and described how everyone uses math in their daily lives. An OLS regression analysis controlling for length of utterance in responses showed that older children (*M* = 2.72, *SD* = 1.20) had significantly higher scale scores than younger children (*M* = 1.87, *SD* = 1.15), *B* = 0.85, $t(101) = 3.68$, $p < .001$, partial $\eta^2 = .118$.

When children were asked to describe how math was used in 10 different activities, the mean math awareness composite score was 1.48 (*SD* = 0.68) on a 0-3 scale, meaning that on average, children were able to indicate that math is included in an activity but either were not able to describe how it is used or described a basic math operation. Activities with the highest math awareness scores were playing with or using money (*M* = 2.17, *SD* = 0.85), keeping score in games or sports (*M* = 2.08, *SD* = 0.92), and playing board games (*M* = 1.82, *SD* = 1.00). Activities with the lowest math awareness scores were playing video games (*M* = 0.88, *SD* =

1.17), playing with puzzles (*M* = 0.92, *SD* = 1.18), and using maps or globes (*M* = 1.02, *SD* = 1.15).

Using OLS regression and controlling for the sum of length of utterances across all ten activities, we found that older children (*M* = 1.74, *SD* = 0.63) had significantly higher math awareness scale scores than younger children (*M* = 1.23, *SD* = 0.64), *B* = 0.30, *t*(101) = 2.90, *p* = .005, partial η^2 = .077. The math uses and math awareness scale scores were combined into the math applicability scale score. After controlling for the sum of length of utterances across both the math uses and the math awareness items, older children (*M* = 4.46, *SD* = 1.57) had significantly higher math applicability scale scores than younger children (*M* = 3.10, *SD* = 1.44), *B* = 1.15, *t*(101) = 3.99, *p* < .001, partial η^2 = .136.

Math utility. The mean math utility composite score was 4.22 out of a possible 5 (*SD* = 0.67) indicating that, on average, children see math as useful and important. An independent samples t-test examined grade group differences. The difference in responses between older children (*M* = 4.27, *SD* = 0.62) and younger children (*M* = 4.17, *SD* = 0.72), *t*(102) = 0.76, *p* = .449, was not significantly different..

Productive disposition. The mean productive composite score was 4.15 (*SD* = 0.79) on a 1-5 scale, indicating that, on average, children see themselves as math users and that working hard in math is worthwhile. Again, an independent samples t-test revealed no significant difference between older children (*M* = 4.15, *SD* = 0.75) and younger children (*M* = 4.15, *SD* = 0.83), *t*(102) = 0.04, *p* = .971.

Overall math utility conceptions composite. To test for grade group differences in children's overall utility conceptions composite, we used an OLS regression model with grade group as the predictor and the sum of length of utterances across all items that feature into the overall math utility composite as a covariate. Older children (*M* = 0.79, *SD* = 2.53) had significantly higher math utility conceptions construct scores than younger children (*M* = -0.73, *SD* = 2.21), *B* = 1.55, *t*(101) = 3.33, *p* = .001, partial η^2 = .099.

Parents' Math Utility Conceptions

Math concepts. Most parents (84%) defined math as number and operations (see Table 6). Unlike their children, however, most parents (62%) also mentioned different math processes, particularly connections (29%) and problem solving (20%). Almost one-third (32%) of parents mentioned geometry as part of their definition of math. The mean math concepts scale score was 2.84 (*SD* = 0.83) on a 0-4 scale. Similar to their children, the most common score on the math concepts scale was a 2 (44%). However, the majority of parents (56%) provided responses that indicated that they defined math as something that is connected to other subjects or daily life activities (3 or 4 on the scale).

Math applicability. When parents were asked to describe how math was used in 10 different activities, the mean math applicability composite score was 2.48 (*SD* = 0.48) on a 0-3 scale, meaning that, on average, parents indicated that math is included in an activity and either described a basic or an advanced math operation. Activities with the highest math applicability scores were using math at a restaurant (*M* = 2.79, *SD* = 0.51), in the kitchen (*M* = 2.73, *SD* = 0.64), and when traveling (*M* = 2.68, *SD* = 0.59). Activities with the lowest math applicability scores were using a cell phone (*M* = 1.97, *SD* = 0.95), making or listening to music (*M* = 2.14, *SD* = 1.03), and playing or watching sports (*M* = 2.33, *SD* = 0.68).

Math utility. The mean math utility composite score was 4.48 (*SD* = 0.40) on a 1-5 scale, indicating that, on average, parents see math as useful and important.

Productive disposition. The mean productive disposition composite score was 3.91 (*SD* = 0.76) on a 1-5 scale, indicating that, on average, parents agree that they are math users and that working hard in math is worthwhile.

Relations Between Parents' and Children's Math Utility Conceptions

To examine the relation between parents' and children's math utility conceptions, we used an OLS regression with parents' math utility conceptions composite score as the predictor, children's grade group and the total length of utterance for all utility conceptions items as covariates (both related to children's overall conceptions scores), and children's math utility conceptions composite score as the outcome. As hypothesized, parents' math utility conceptions significantly predicted children's math utility conceptions, $\beta = .194$, $t(98) = 2.10$, $p = .038$. Child grade group, $\beta = .219$, $t(98) = 2.35$, $p = .021$, and length of utterance, $\beta = .244$, $t(98) = 2.60$, $p = .011$, were both significant covariates. This suggests that if parents have the knowledge and beliefs that math is useful for daily lives, their children tend to have this knowledge and these beliefs as well.

We examined whether the relation between parents' and children's math utility conceptions was moderated by three parent math socialization practices: the percent of children's math homework time that parents help with math homework, parent-reported children's math home engagement, and the extent to which parents report that their children see them engaging in math activities. We used one OLS regression to test these effects, with parents' overall conceptions score, the three home numeracy environment variables, the three interaction terms between parents' conceptions and each of the home environment variables as predictors, children's grade group and the sum of math conception items length of utterances as covariates, and children's math utility conceptions as the outcome (see Table 6). Parent's math utility conceptions, $p = .013$, remained a significant predictor of children's math utility conceptions. Contrary to our hypothesis, the percentage of time that parents help with math homework was not a significant moderator of the relation between parents' and children's math utility conceptions, $p = .321$. However, parent-reported mean frequency of children's home math engagement, $p = .025$, and the frequency with which parents report their children see them doing math, $p = .022$, significantly moderated the relation between parents' and children's math utility conceptions.

We probed the interactions of frequency of children's home math engagement and how often children see their parents doing math on the relation between parents' and children's math utility conceptions using a simple slopes analysis (Cohen, Cohen, West & Aiken, 2003; Hayes & Rockwood, 2017; Preacher, Curran, & Bauer, 2006). For both moderators, we probed the relations between parents' and children's math utility conceptions at the mean and at one standard deviation above and one below the mean of each moderator, holding all other variables constant at their respective mean values. For home math engagement, as the frequency of children's home math engagement decreases, the strength of the relation between parents' and children's math utility conceptions increases (see Figure 2). On the contrary, as the frequency with which children see their parents doing math increases, the strength of the relation between parents' and children's math utility conceptions increases (see Figure 3).

Relations Between Children's Home Math Engagement and Math Utility Conceptions

To explore the relations between children's home math engagement, five separate OSL regressions were conducted with child-reported frequency of home math engagement as the predictor, grade group and the relevant length of utterance, as appropriate, as covariates, and overall children's math utility conceptions composite score or one of the four components of math utility conceptions as the outcome. Consistent with Metzger et al. (2018, Study 1), overall home math engagement was not a significant predictor of overall children's math utility conceptions, $\beta = .083$, $t(98) = 0.91$, $p = .368$. Home math engagement also did not significantly predict children's math concepts, $\beta = -.022$, $t(100) = -0.25$, $p = .807$, math applicability, $\beta = -1.32$, $t(100) = -1.32$, $p = .190$, and math utility scale scores, $\beta = .132$, $t(101) = 1.34$, $p = .183$. However, home math engagement was a significant predictor of children's productive disposition scale scores, $\beta = .243$, $t(101) = 2.51$, $p = .014$.

Discussion

This study examined whether parents' math utility conceptions and home math engagement were associated with children's math utility conceptions. Consistent with Metzger et al. (2018, Study 1), children's knowledge about math was heavily focused on math operations and as something that is learned and used in school. Children had limited knowledge of how math featured into their daily lives. They were able, on average, to identify that math was used in activities but not always how it was used. In contrast, and perhaps not surprisingly, parents were far more likely to describe math processes as part of their definitions and, on average, were able to describe a basic or advanced aspect of math that is featured in activities.

There were three particularly noteworthy findings. First, parents' math utility conceptions significantly predicted children's math utility conceptions. This finding may point to the home as a setting for potential intervention. Prior research with children in high school showed that parent-based interventions have successfully increased both parents' and children's math utility beliefs and the level of math courses in which children enroll (Harackiewicz et al., 2012; Rozek et al., 2017). Parents and children in this study exhibited a range of math utility knowledge and beliefs about math utility. Results suggest that interventions geared towards teaching parents about math utility for elementary-age children may lead to an improvement in children's knowledge and beliefs about math utility.

Second, the relation between parents' and children's math utility conceptions was moderated by children's home math engagement and the frequency with which children see their parents doing math. Parents' conceptions of math utility were associated more strongly with children's conceptions for children who engage less frequently in home-based math activities. We know that when children engage in math activities at home, they develop an understanding of how math is incorporated into their world (Clements & Sarama, 2007; Ginsburg et al., 2008; Missall et al., 2015). Perhaps when children are engaging less frequently in math activities, their conceptions about how math is used in daily life primarily comes from what they hear their parents say about math. Essentially, children may identify more with their parents' conceptions about the usefulness of math when they have less direct opportunity to experience the ways in which math is used in daily activities.

Parent's and children's math utility conceptions were also associated more highly for children who more frequently see their parents using math. This may be because parents are using opportunities to model the ways in which math is useful in daily life. The extent to which children see their parents using math is also associated with children's early math skills (Sonnenschein et al., 2016). Results from the current study suggest that this role modeling behavior may be important for the development of math utility conceptions as well. Children may internalize the ways in which parents talk about math they are using during these activities in a way that is different from when parents simply tell children about how math can be used. Also, if parents are using more math language with their children during these activities, this may improve children's math language vocabulary, which, in turn, improves children's math skills (Purpura, Napoli, Wehrspann, & Gold, 2017; Purpura & Reid, 2016). We did not specifically ask parents whether they engage their children in conversation while they doing these activities, so it unclear whether this effect is due to parents discussing math uses with children or from children just observing their parents' behaviors.

Maloney et al.'s (2015) research suggests that the more parents and children engage in math discussions together, specifically during homework time, the more likely they are to pass on their beliefs about math to their children. This study supports that notion, but results suggest that for math utility conceptions, the contexts in which parents pass on their knowledge and beliefs about the usefulness of math are different than for math anxiety. For math utility conceptions, this "transfer" of knowledge may occur primarily when parents are demonstrating their own uses of math in their daily lives and/or in the absence of children learning during their own home-based math activities.

Third, older children demonstrated higher math concepts and math applicability knowledge than younger children, after controlling for potential developmental differences in language skills, but there were no differences between the two groups on math utility or productive disposition. This distinction may be important for understanding the differences in the development of math utility knowledge and beliefs. These findings suggest that as children move through early elementary school, they are acquiring more knowledge about the utility of math through experiences at home and in school. However, consistent with past research on subjective task value (Eccles et al, 1993; Gottfried, Marcoulides, Gottfried, Oliver, & Guerin, 2007; Wigfield et al., 1997), children's beliefs that math is useful and their effort in math is worthwhile may remain stable during this time. The lack of significant differences in math utility beliefs at this age is consistent with other research that shows that beliefs about math utility do not typically begin to decline until late elementary school or middle school (e.g., Eccles et al., 1993; Musu-Gillette et al., 2015). This may indicate that there is a different developmental trajectory for knowledge and beliefs about math utility.

Implications for Practice

The findings from this study indicate that parents' math utility conceptions, may be important for the development of children's math utility conceptions. Parents have many opportunities to demonstrate their math utility knowledge and beliefs to their children. Research shows that the extent to which parents use number talk at home relates to children's number knowledge (Gunderson & Levine, 2011; LeFevre et al., 2009; Levine et al., 2010; Ramani et al., 2015), so there may be a similar effect if parents are encouraged to share their knowledge and beliefs about the usefulness of math. If parents know that what they say to their children about math can influence how they feel about math, especially when they model their own math behaviors for their children, it may motivate them to better understand how math is useful in daily life and how to teach this to their children.

Limitations and Future Directions

Although this study offered new insight into how parents' math utility conceptions may be associated with their children's conceptions, there are three important limitations to consider. One, we may not have sufficiently tapped into home math experiences. Although we used a variety of activities, there may be more interactions or informal math discussions that we did not measure. Home math engagement was related to children's productive disposition, but not to other aspects of math utility conceptions. This may be because our rating scale did not account for the extent to which children engage in the math-related aspects of the activities in the questionnaire.

Two, although the sample was relatively racially/ethnically diverse, the majority of the sample were families with highly-educated parents in middle- to upper middle- income households. This means that findings may not be generalizable to all demographic groups. Additionally, because there were not enough participants in each of the demographic groups, we could not test for differences between racial/ethnic groups or families with different socioeconomic statuses. This is important to consider because there may be differences in these relations for families that are dissimilar to those in this sample. Although current research does not suggest specific ways in which these relations may be different, future research is needed with different socioeconomic groups to determine whether these results are generalizable to a broader population.

Three, children's responses may have been limited by their verbal abilities and may not capture their complete math utility conceptions. However, the coding categories were developed based on content of responses, rather than linguistic sophistication of children's responses. We also controlled for length of utterance to mitigate some of the potential developmental differences in how much children articulate in a given response. In addition, research shows that children as young as preschool-age can respond effectively to both open- and closed-ended questions about math self-concepts and home math engagement (Eccles et al., 1993; Marsh, Ellis, & Craven, 2002; Mazzocco et al., 2012; Ramani & Siegler, 2008; Wigfield et al., 1997).

Despite these limitations, this study offers new information to increase our understanding of children's math utility conceptions and how they develop. This study also points to important next steps in future research about math utility conceptions. Future research should focus on understanding how children's math utility conceptions relate to children's math achievement. Research that has looked at the relation between math utility conceptions and achievement (e.g., Mazzocco et al., 2012) is fairly limited in scope. Using the conceptual model and related measures from this study would extend current research and help us better understand what aspects of math utility are most associated with math achievement.

More work is needed to understand how to create effective home-based interventions to increase children's knowledge about the usefulness of math in their daily lives. Some of this work has been done with older children (e.g., Harackiewicz et al., 2012), but not with children as young as early elementary school. The long-term effects of home-based math utility interventions needs to be studied to determine if improving math utility conceptions early may buffer some of the typical decline in math utility beliefs that children experience in older grades. If interventions can be adapted for math that children are learning in every grade level, their knowledge and beliefs about math utility may continue to improve over time.

Conclusion

The primary goal of this study was to examine the relation between parents' and children's math utility conceptions. This study supports the notion that parents play an important role in the development of children's knowledge and beliefs about the usefulness of math. These results may guide home-based interventions for early elementary-aged children to improve their understanding of the importance of math in our daily lives.

References

Baker, L., & Scher, D. (2002). Beginning reader's motivation for reading in relation to parental beliefs and home reading experiences. *Reading Psychology, 23*, 239-269. doi:10.1080/713775283

Barbarin, O. A., Early, D., Clifford, R., Bryant, D., Frome, P., Burchinal, M., ... & Pianta, R. (2008). Parental conceptions of school readiness: Relation to ethnicity, socioeconomic status, and children's skills. *Early Education and Development*, *19*, 671-701. doi:10.1080/10409280802375257

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting, 5,* 35-45. doi:10.1002/(SICI)1099-0917(199603)5:1<35::AID-EDP113>3.0.CO;2-0

Chouinard, R., Karsenti, T., & Roy, N. (2007). Relations among competence beliefs, utility value, achievement goals, and effort in mathematics. *British Journal of Educational Psychology*, *77,* 501-517. doi:10.1348/000709906X133589

Clements, D. H., & Sarama, J. (2006). Your child’s mathematical mind. *Scholastic Parent & Child, 14*, 30-39.

Clements, D. H., & Sarama, J. (2007). Early childhood mathematics learning. In F. K. Lester Jr. (Vol. Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics, Vol. 1* (pp.461-566). Charlotte, NC: Information Age Publishing.

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach (2nd ed.).* New York: Routledge.

Cohen, J. (1960). A coefficient of agreement for nominal scales. *Educational and Psychological Measurement*, *20*, 37-46.

Cohen, J., Cohen, P., West, S. G., & Aiken, L. S. (2013). *Applied multiple regression/correlation analysis for the behavioral sciences*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Downer, J. T., & Pianta, R. C. (2006). Academic and cognitive functioning in first grade: Associations with earlier home and child care predictors and with concurrent home and classroom experiences. *Social Psychology Review, 35*, 11-30.

Dunlosky, J., & Rawson, K. A. (2012). Overconfidence produces underachievement: Inaccurate self evaluations undermine students' learning and retention. *Learning and Instruction*, *22*, 271-280. doi:10.1016/j.learninstruc.2011.08.003

Eccles, J. S. (2009). Who am I and what am I going to do with my life? Personal and collective identities as motivators of action. *Educational Psychologist, 44*, 78–89. doi:10.1080/00461520902832368

Eccles, J. S., & Wigfield, A. (2002). Motivational beliefs, values, and goals. *Annual Review of Psychology, 53*, 109-132. doi:10.1146/annurev.psych.53.100901.135153

Eccles, J. S., Wigfield, A., Harold, R., & Blumenfeld, P. (1993). Age and gender differences in children's self-and task perceptions during elementary school. *Child Development, 64*, 830-847. doi:10.1111/j.1467-8624.1993.tb02946.x

Fleiss, J. L. (1981). *Statistical methods for rates and proportions (2nd ed.).* New York: John Wiley.

Geary, D. C. (2006). Development of mathematical understanding. In W. Damon & R. M. Lerner (Series Eds.) & D. Kuhn & R. Siegler (Vol. Eds.), *Handbook of child psychology: Vol. 2 Cognition, perception, and language* (6^{th} ed., pp. 777-810). New York: Wiley.

Ginsburg, H. P., Lee, J. S., Boyd, J. S. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report*, *22,* 3-22.

Gniewosz, B., & Noack, P. (2012). What you see is what you get: The role of early adolescents' perceptions in the intergenerational transmission. *Educational Psychology, 37*, 70-79. doi:10.1016/j.cedpsych.2011.10.002

Gottfried, A. E., Marcoulides, G. A., Gottfried, A. W., Oliver, P. H., & Guerin, D. W. (2007). Multivariate latent change modeling of developmental decline in academic intrinsic math motivation and achievement: Childhood through adolescence. *International Journal of Behavioral Development*, *31*, 317-327. doi:10.1177/0165025407077752

Gunderson, E. A., & Levine, S. C. (2011). Some types of parent number talk count more than others: Relations between parents' input and children's cardinal-number knowledge. *Developmental science*, *14*, 1021-1032. doi:10.1111/j.1467-7687.2011.01050.x

Guo, J., Marsh, H. W., Parker, P. D., Morin, A. J., & Yeung, A. S. (2015). Expectancy-value in mathematics, gender and socioeconomic background as predictors of achievement and aspirations: A multi-cohort study. *Learning and Individual Differences*, *37*, 161-168. doi:10.1016/j.lindif.2015.01.008

Harackiewicz, J. M., Rozek, C. S., Hulleman, C. S., & Hyde, J. S. (2012). Helping parents to motivate adolescents in mathematics and science: An experimental test of a utility-value intervention. *Psychological Science, 23,* 899-906. doi:10.1177/0956797611435530

Hayes, A. F., & Rockwood, N. J. (2017). Regression-based statistical mediation and moderation analysis in clinical research: Observations, recommendations, and implementation. *Behaviour Research and Therapy*, *98,* 39-57. doi:10.1016/j.brat.2016.11.001

Hoff, E. (2014). *Language development*. Belmont, CA: Wadsworth.

Hulleman, C. S., Kosovich, J. J., Barron, K. E., & Daniel, D. B. (2017). Making connections: Replicating and extending the utility value intervention in the classroom. *Journal of Educational Psychology*, *109*, 387. doi:10.1037/edu0000146

Huntsinger, C.S., Jose, P.E., Larson, S.L. Balsink Krieg, D. and Shaligram. C. (2000). Mathematics, vocabulary, and reading development in Chinese American and European American children over the primary school years. *Journal of Educational Psychology 92*, 745–60. doi:10.1037//0022-0663.92.4.745

Jacobs, J. E., Lanza, S., Osgood, D. W., Eccles, J. S., & Wigfield, A. (2002). Changes in children's self-competence and values: Gender and domain differences across grades one through twelve. *Child Development*, *73,* 509-527. doi:10.1111/1467-8624.00421

Jansen, A. (2012). Developing productive dispositions during small-group work in two sixth-grade mathematics classrooms: Teachers' facilitation effort and students' self-reported benefits. *Middle Grades Research Journal, 7,* 37-56.

Jodl, K. M., Michael, A., Malanchuk, O., Eccles, J. S., & Sameroff, A. (2001). Parents' roles in shaping early adolescents' occupational aspirations. *Child Development*, *72,* 1247-1266. doi:10.1111/1467-8624.00345

King, R. B., & McInerney, D. M. (2014). Mapping changes in students' English and math self-concepts: a latent growth model study. *Educational Psychology*. doi:10.1080/01443410.2014.909009

Koo, T. K., & Li, M. Y. (2016). A guideline of selecting and reporting intraclass correlation coefficients for reliability research. *Journal of Chiropractic Medicine, 2016,* doi:10.1016/j.jcm.2016.02.012.

Landis, J. R. & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics*, *33*, 159–174.

Laski, E. V., & Siegler, R. S. (2014). Learning from number board games: You learn what you encode. *Developmental Psychology*, *50*, 853-864. doi:10.1037/a0034321

Lazarides, R., Harackiewicz, J., Pesu, L., & Viljaranta, J. (2015). The role of parents in students' motivational beliefs and values. In C. M. Rubie-Davies, J. M. Stephens & P. Watson (Eds.), *The social psychology of the classroom international handbook* (pp. 53-66). London: Routledge.

LeFevre, J.-A., Polyzoi, E., Skwarchuk, S.-L., Fast, L., & Sowinski, C. (2010). Do home numeracy and literacy practices of Greek and Canadian parents predict the numeracy skills of kindergarten children? *International Journal of Early Years Education, 18,* 55-70. doi:10.1080/09669761003693926

LeFevre, J.-A., Skwarchuk, S.-L., Smith-Chant, B.L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children's math performance in the early school years. *Canadian Journal of Behavioral Science, 41,* 55-66. doi:10.1037/a0014532

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology, 46,* 1309-1319. doi:10.1037/a0014532

Maloney, E. A., Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2015). Intergenerational effects of parents' math anxiety on children's math achievement and anxiety. *Psychological Science, 2015,* doi:10.1177/0956797615592630.

Marsh, H. W., Ellis, L. A., & Craven, R. G. (2002). How do preschool children feel about themselves? Unraveling measurement and multidimensional self-concept structure. *Developmental psychology*, *38*, 376-393. doi:10.1037//0012-1649.38.3.376

Marsh, H. W., & Martin, A. J. (2011). Academic self-concept and academic achievement: Relations and causal ordering. *British Journal of Educational Psychology*, *81*, 59-77. doi:10.1348/000709910X503501

Mazzocco, M. M., Hanich, L. B., & Noeder, M. M. (2012). Primary school age students' spontaneous comments about math reveal emerging dispositions linked to later mathematics achievement. *Child Development Research*, *2012*, doi:10.1155/2012/170310.

McGraw, K. O., & Wong, S. P. (1996). Forming inferences about some intraclass correlation coefficients. *Psychological methods*, *1*, 30-46.

Metzger, S. R., Sonnenschein, S., & Galindo, C. (2018). *Elementary-age children's conceptions about math utility and their home-based math engagement.* Manuscript submitted for publication.

Missall, K., Hojnoski, R. L., Caskie, G. I., & Repasky, P. (2015). Home numeracy environments of preschoolers: Examining relations among mathematical activities, parent mathematical beliefs, and early mathematical skills. *Early Education and Development*, *26*, 356-376. doi:10.1080/10409289.2015.968243

Mitchell, T. (1999). Changing student attitudes towards mathematics. *Primary Educator*, *5*(4), 2-7.

Muenks, K., Wigfield, A., & Eccles, J. S. (2018). I can do this! The development and calibration of children's expectations for success and competence beliefs. *Developmental Review*. doi:10.1016/j.dr.2018.04.001

Muis, K. R. (2004). Personal epistemology and mathematics: A critical review and synthesis of research. *Review of Educational Research, 74*, 317-377. doi:10.3012/00346543074003317

Musu-Gillette, L. E., Wigfield, A., Harring, J. R., & Eccles, J. S. (2015). Trajectories of change in students’ self-concepts of ability and values in math and college major choice. *Educational Research and Evaluation*, *21*, 343-370. doi:10.1080/13803611.2015.1057161

Nagy, G., Watt, H. M., Eccles, J. S., Trautwein, U., Lüdtke, O., & Baumert, J. (2010). The development of students' mathematics self-concept in relation to gender: Different countries, different trajectories? *Journal of Research on Adolescence*, *20*, 482-506. doi:10.1111/j.1532-7795.2010.00644.x

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences of Education. Washington, DC: National Academy Press.

National Mathematics Advisory Panel (2008). Foundations for success: The final report of the National Mathematics Advisory Panel.

Okagaki L. & Bingham, G. E. (2005). In T. Luster & L. Okagaki (Eds.). *Parenting: An Ecological Perspective* (2nd Ed., pp. 3-70). Mahwah, NJ: Laurence Erlbaum Associates.

Okagaki, L., & Sternberg, R. J. (1993). Parental beliefs and children's school performance. *Child development*, *64*, 36-56. doi:10.2307/1131436

Papadakis, S., Kalogiannakis, M., & Zaranis, N. (2017). Improving mathematics teaching in kindergarten with realistic mathematical education. *Early Childhood Education Journal*, *45*, 369-378. doi:10.1007/s10643-015-0768-4

Parsons [Eccles], J. E., Adler, T., & Kaczala, C. (1982). Socialization of achievement attitudes and beliefs: Parental influences. *Child Development, 53*, 310-321. doi: 10.2307/1128973

Perlmutter, J., Bloom, L., Rose, T., & Rogers, A. (1997). Who uses math? Primary children's perceptions of the uses of mathematics. *Journal of Research in Childhood Education, 12*, 58-70.

Preacher, K. J., Curran, P. J., & Bauer, D. J. (2006). Computational tools for probing interactions in multiple linear regression, multilevel modeling, and latent curve analysis. *Journal of Educational and Behavioral Statistics*, *31*, 437-448. doi: 10.3102/10769986031004437

Purpura, D. J., Napoli, A. R., Wehrspann, E. A., & Gold, Z. S. (2017). Causal connections between mathematical language and mathematical knowledge: A dialogic reading intervention. *Journal of Research on Educational Effectiveness*, *10*, 116-137. doi:10.1080/19345747.2016.1204639

Purpura, D. J., & Reid, E. E. (2016). Mathematics and language: Individual and group differences in mathematical language skills in young children. *Early Childhood Research Quarterly*, *36*, 259-268. doi:10.1016/j.ecresq.2015.12.020

Ramani, G. B., Rowe, M. L., Eason, S. H., & Leech, K. A. (2015). Math talk during informal learning activities in Head Start families. *Cognitive Development*, *35*, 15–33. doi:10.1016/j.cogdev.2014.11.002

Ramani, G. B & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development, 79,* 375-394. doi:10.1111/j.1467-8624.2007.01131

Rittle-Johnson, B. (2017). Developing mathematics knowledge. *Child Development Perspectives, 11*, 184-190. doi:10.1111/cdep.12229

Rozek, C. S., Hyde, J. S., Svoboda, R. C., Hulleman, C. S., & Harackiewicz, J. M. (2015). Gender differences in the effects of a utility-value intervention to help parents motivate adolescents in mathematics and science. *Journal of Educational Psychology*, *107*, 195-206. doi:10.1037a0036981

Sarama, J. & Clements, D. H. (2006). 5 math myths. *Scholastic Parent & Child*, *13*, 44-45.

Sarama, J. & Clements, D. H. (2007). How children problem solve. *Early Childhood Today*, *21*, 16-19.

Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processes in early number development. *Monographs of the Society for Research in Child Development, 52,* 153–159. doi:10.2307/1166071

Schofield, T. J., & Weaver, J. M. (2015). Democratic parenting beliefs and observed parental sensitivity: Reciprocal influences between coparents. *Journal of Family Psychology,* advanced online publication. dx.doi.org/10.1037/fam0000166

Siegler, R. S., & Mu, Y. (2008). Chinese children excel on novel mathematics problems even before elementary school. *Psychological Science, 19,* 759-763. doi:10.1111/j.1467-9280.2008.02153.x

Simons, C. L., Metzger, S. R., & Sonnenschein, S. (2018). *Children's perceptions of key executive function processes: What they are and how children learn to use them.* Manuscript submitted for publication.

Simpkins, S. D., Fredricks, J. A., & Eccles, J. S. (2012). Charting the Eccles' expectancy-value model from mothers' beliefs in childhood to youths' activities in adolescence. *Developmental Psychology*, *48*, 1019-1032. doi:10.1037/a0027468

Simpkins, S. D., Fredricks, J. A., & Eccles, J. S. (2015). Parent beliefs to youth choices: Mapping the sequence of predictors from childhood to adolescence. *Monographs of the Society for Research in Child Development Research, 80*, 1-169.

Singh, K., Granville, M., & Dika, S. (2002). Mathematics and science achievement: Effects of motivation, interest, and academic engagement. *The Journal of Educational Research*, *95*, 323-332. doi:10.1080/00220670209596607

Skwarchuk, S.-L., & LeFevre, J.-A. (2015). The role of the home environment in children's early numeracy development: A Canadian perspective. In B. Perry, A. MacDonald, & A. Gervasconi (Eds.), *Mathematics and transitions to school: International perspectives* (pp. 103-117). Singapore: Springer.

Sobel, D. M., & Letourneau, S. M. (2015). Children's developing understanding of what and how they learn. *Journal of experimental child psychology*, *132*, 221-229. doi:10.1016/j.jecp.2015.01.004

Sonnenschein, S., Baker, L., & Garrett, A. (2004, December). Final Report of the 2004 Super Kids Camp. Sponsored by Parks and People Foundation.

Sonnenschein, S., Galindo, C., Metzger, S. R., Thompson, J. A., Huang, H. C., & Lewis, H. (2012). Parents' beliefs about children's math development and children's participation in math activities. *Child Development Research, 2012,* doi:10.1155/2012/851657.

Sonnenschein, S., Galindo, C., Simons, C. L., Metzger, S. R., Thompson, J. A., & Chung, M. (2018). How do children learn mathematics? Chinese and Latino immigrant perspectives. In S. S. Chuang, & C. L. Costigan (Eds.), *Parental roles and relationships in immigrant families: An international approach.* New York, NY: Springer Science and Business Media

Sonnenschein, S., Metzger, S. R., & Thompson, J., A. (2016). Low-income parents' socialization of their preschoolers' early reading and math skills. *Research in Human Development, 13,* 207-224. doi: 10.1080/15427609.2016.1194707

U.S. Department of Education. (2014). *2013 Grade 12 Reading and Mathematics Results: National Assessment of Education Progress.* National Center for Educational Statistics.

U.S. Department of Education. (2015a). *National Assessment of Education Progress.* National Center for Educational Statistics.

U.S. Department of Education. (2015b). *Trends in International Math and Science Study*. National Center for Educational Statistics.

U.S. Department of Education. (2018). *National Assessment of Education Progress.* National Center for Educational Statistics.

Viljaranta, J., Lazarides, R., Aunola, K., Räikkönen, E., & Nurmi, J.-E. (2015). The different role of mothers' and fathers' beliefs in the development of adolescents' mathematics and literacy task values. *International Journal of Gender, Science and Technology, 7,* 297-317.

Vo, V. A., Li, R., Kornell, N., Pouget, A., & Cantlon, J. F. (2014). Young children bet on their numerical skills metacognition in the numerical domain. *Psychological Science, 25*, 1712-1721. doi:10.1177/0956797614538458

Wigfield, A. & Eccles, J. S. (2000). Expectancy-value theory of achievement motivation. *Contemporary Educational Psychology, 25*, 68-81. doi:10.1006/ceps.1999.1015

Wigfield, A., Eccles, J. S., Yoon, K. S., Harold, R. D, Arbreton, A. J. A., Freedman-Doan, C. & Blumenfeld, P. C. (1997). Change in children's competence beliefs and subjective task values across the elementary school years: A 3-year study. *Journal of Educational Psychology, 89*, 451-469. doi:10.1037/0022-0663.89.3.451

Wigfield, A., & Guthrie, J. T. (1997). Relations of children's motivation for reading to the amount and breadth or their reading. *Journal of Educational Psychology*, *89*, 420-432. doi:10.1037/0022-0663.89.3.420

Table 1

Description of Measures

Constructs	Item	Codes or Scale
Knowledge about Math		
Math Concepts	"What is math?"	Content Number and operations *("math is numbers"*) Algebra (*"like when 7 minus X equals 2"*) Geometry (*"math is shapes"*) Measurement (*"we use rulers to see how many inches the eraser is"*) Data analysis and probability (*"when you make charts with everyone's eye color in the class"*) Processes Problem solving (*"math is solving problems"*) Reasoning and proof (*"logic is a part of math"*) Communication (*"when the teacher tells us to tell how we got our answer"*) Connections (*"math is in science and physics too"*) Representations (*"we use base 10 blocks to find the answers"*) Affect Positive (*"math is fun activities"*) Negative (*"math is something I hate to do"*) Difficulty Easy (*"math is a bunch of easy stuff"*) Hard (*"it's the hard stuff"*) Math Makes you Smarter (*"math is how people get smarter"*)
	Math Concepts Score Adapted from Mazzocco et al. (2012)	0 No response, circular response, a response unrelated to math, or another uninformative response (*"I don't know"*) 1 Response is unspecific or only indirectly related to math as a primary school subject. The response may include references to activities performed in relation to math but with no discernable reference to math concepts (*"how people get smarter…help people learn"*) 2 Unelaborated basic concepts or mechanics of math, including numbers, operations, math

		problems, or learning math (*“math is adding and subtracting”*) 3 Elaborate concepts of math that may feature some application of math to solving problems or answering questions, including description of how math is used in other subjects (*“math is a way, like if you had like a certain amount of money…and you need to know how much change you would get back”*) 4 Concept of math as a useful tool and/or discussion about how math is important/critical/part of daily life (*“math keeps everything together. If there was no math, buildings could not stay up…we could not figure out problems”*)
Applicability of Math	How do you learn math?	School (*“I learn at school,” “from my teacher,”* or *“by doing your homework”*) Home (*“with my daddy,” “my parents help me with my homework,”* or *“playing math games on the computer with mommy”*) Practicing and/or by Doing Math (*“you have to practice it”*) Hierarchical Learning (*“you start with the easy stuff and then it gets harder and harder”*) Specific activities that are not linked to home or school (“*we learn math by using blocks”*) Friends/siblings (*“my sister helps me learn”*) Self-taught (*“I read a book and teach myself”*)
	Who uses math?	Teachers (*“my teacher”*) Parents (*“my mom and dad”*) Children (*“students…I use it too”*) Other adults (*“scientists…engineers”*) Everyone (*“everybody uses it”*)
	Math Uses Score (who uses math? and how does {person} use math?)	0 No response, circular response, a response unrelated to math, or another uninformative response (*“I cannot think of anyone”*) 1 Child mentioned one or two people/professions that use math but cannot really describe how they use math in detail or just discuss that they use math as part of math education. This includes teachers who only use math to

		teach children, not as a tool for their lives (*“scientists- they use it like with needing to know numbers”* or *“teachers- they use it to teach kids math”*)
	2	Child said that everyone uses math but can only describe it as part of their education (*“everyone- everybody has to learn math in school”*)
	3	Child mentioned one or two people/professions that use math and elaborated on how they use math in their jobs and/or lives (*“scientists- they have to measure different amounts of liquids and use formulas to solve problems”* or *“teachers- they have to use it to figure out with percents with grading or counting the number of kids who are absent that day*)
	4	Child said that everyone uses math and describes how you need it in jobs and/or daily life (*“everyone- everyone has to use math in their life all the time…like for solving problems and at the grocery store”*)
Math Awareness Score (Do you think that math is used when you do (activity)? How is math used in (activity)?)	0	Child did not identify that math was used or if s/he said that math was used, but the description was not related to math (*“you read the instructions on the card”*).
	1	Child said that math was used in the activity but did not describe how (*“I don’t know”*).
	2	Child described a basic math skill, such as number recognition (*“when you play cards, there are numbers on them”*) or operation, such as counting (*“you count the spaces when you roll the die”*)
	3	Child described an advanced math skill, such as number magnitude comparison (*“when you keep score, you have to know whose score is bigger so you know who won”*) or measurement (*“for the recipe, you have to measure ¼ cup of flour and 2 cups of sugar”*)

Beliefs about Math

Utility Value	Math is useful to me for things other than school. Math is useful to everyone. I think people use math all the time in their lives. Math is more useful than other subjects. I need to learn math to do well in school. It is important for me to learn math. My parents think it is important for me to learn math. I think it is important for everyone to learn math. It is important for me to do well in math.	1 “Not at all like me” 2 3 “A little like me” 4 5 “A lot like me”
Productive Disposition	Math is a subject I can use in my life. Learning math is worth my time. Working hard in math is worth my time. Working hard in math helps me do better in math. I am good at math. I am better at math than my classmates. I am better at math than my other subjects. I know that I will do well in math this year. Math is easy for me. When doing math is difficult for me, working harder helps me solve it.	1 “Not at all like me” 2 3 “A little like me” 4 5 “A lot like me”
Home Math Engagement		
	How often do you (activity) at home? (Example activities: play board games, play card games, help with cooking)	1 “Never/Almost never” 2 3 “Sometimes” 4 5 “Every day/Almost every day”

Table 2

Children's Responses to "What is Math?" by Grade Group

Coding Category	Overall *N* = 104	Rising 1st/2nd Grades *N* = 54	Rising 3rd/4th Grades *N* = 50
Content	92.3%	88.9%	96.0%
Number and Operations	89.4%	87.0%	92.0%
Counting	6.7%	5.6%	8.0%
Number Knowledge	8.7%	5.6%	12.0%
Number Transformations	73.1%	64.8%	82.0%
Estimation	1.9%	0.0%	4.0%
Number Patterns	1.9%	3.7%	0.0%
Algebra	11.5%	13.0%	10.0%
Geometry	12.5%	11.1%	14.0%
Measurement	6.7%	7.4%	6.0%
Data Analysis and Probability	0.0%	0.0%	0.0%
Processes	20.2%	14.8%	26.0%
Problem Solving	4.8%	3.7%	6.0%
Reasoning and Proof	0.0%	0.0%	0.0%
Communication	0.0%	0.0%	0.0%
Connections	12.5%	3.7%	22.0%
Representations	4.8%	7.4%	2.0%
Affect	10.6%	14.8%	6.0%
Positive	10.6%	14.8%	6.0%
Negative	1.0%	1.9%	0.0%
Difficulty	7.7%	7.4%	8.0%
Easy	1.9%	3.7%	0.0%
Hard	6.7%	7.4%	6.0%
Math Makes you Smarter	7.8%	11.1%	4.0%
Math Knowledge Scale Score (*M* (*SD*))	2.11 (0.75)	1.91 (0.56)	2.32 (0.87)

Note. Percentages do not add to 100%, because children could mention more than one coding category in their response.

Table 3

Children's Responses to "How Do You Learn Math?"

	Overall N = 104	Rising 1st/2nd Grades N = 54	Rising 3rd/4th Grades N = 50
School or School-Related Activities	87.5%	83.3%	92.0%
Teachers Teach	71.2%	70.4%	72.0%
Home Math Activities	38.5%	37.0%	40.0%
Parents Teach	33.7%	33.3%	34.0%
Practicing and/or by Doing Math	11.5%	11.1%	12.0%
Hierarchical Learning	5.8%	7.4%	4.0%
Specific Activities (not linked to home/school)	16.3%	14.8%	18.0%
Friends/Siblings	7.7%	9.3%	6.0%
Self-Taught	8.7%	5.6%	12.0%

Note. Percentages do not add to 100%, because children could mention more than one coding category in their response.

Table 4

Who Uses Math? Responses by Grade

Coding Category	Overall *N* = 104	Rising 1st/2nd Grades *N* = 54	Rising 3rd/4th Grades *N* = 50
Teachers	30.8%	29.6%	32.0%
Parents	15.4%	14.8%	16.0%
Children	46.2%	55.6%	36.0%
Other Adults[a]	38.5%	31.5%	46.0%
Everyone	45.2%	37.0%	54.0%
Math Uses Scale Score (*M* (*SD*))	2.28 (1.24)	1.87 (1.15)	2.72 (1.20)

Note. Percentages do not add to 100%, because children could mention more than one coding category in their response.

[a]The most commonly mentioned "other adults" were scientists, adult non-parent relatives, mathematicians, engineers, carpenters/builders/architects, inventors, doctors, the primary investigator of this study, and cashiers.

Table 5

Parents' Responses to "What is Math?" by Children's Grade Group

Coding Category	Overall *N* = 94*	Parents of Rising 1st/2nd Graders *N* = 54	Parents of Rising 3rd/4th Graders *N* = 48
Content	86.2%	81.5%	91.7%
Number and Operations	84.0%	77.8%	89.6%
Counting	1.1%	0.0%	2.1%
Number Knowledge	6.4%	3.7%	8.3%
Number Transformations	23.4%	18.5%	29.2%
Estimation	0.0%	0.0%	0.0%
Number Patterns	18.1%	9.3%	25.0%
Algebra	14.9%	18.5%	12.5%
Geometry	31.9%	31.5%	31.3%
Measurement	14.9%	14.8%	12.5%
Data Analysis and Probability	1.1%	0.0%	2.1%
Processes	61.7%	64.8%	54.2%
Problem Solving	20.2%	20.4%	16.7%
Reasoning and Proof	12.8%	9.3%	16.7%
Communication	9.6%	13.0%	6.3%
Connections	28.7%	33.3%	22.9%
Representations	4.3%	5.6%	2.1%

Note. Percentages do not add to 100%, because parents could mention more than one coding category in their response.

*These percentages only reflect percentage of unique parent participants.

Table 6

Moderation Regression for Parents' Math Utility Conceptions Predicting Children's Math Utility Conceptions

	b	*t*	*p*
Parents' math utility conceptions	0.268	2.53	.013
Percent of time parents help with homework	-0.209	-0.35	.730
Parents' conceptions X homework help	0.274	1.00	.321
Children's math engagement	0.771	1.69	.095
Parents' conceptions X children's engagement	-0.467	-2.28	.025
How often child sees parents doing math	-0.355	-1.57	.121
Parents' conceptions X child sees parents	0.187	2.34	.022
Child Grade Group	1.053	2.33	.022
Length of utterance for all items in overall conceptions	0.003	2.73	.008

Note. df = 90. Adjusted R^2 = .25.

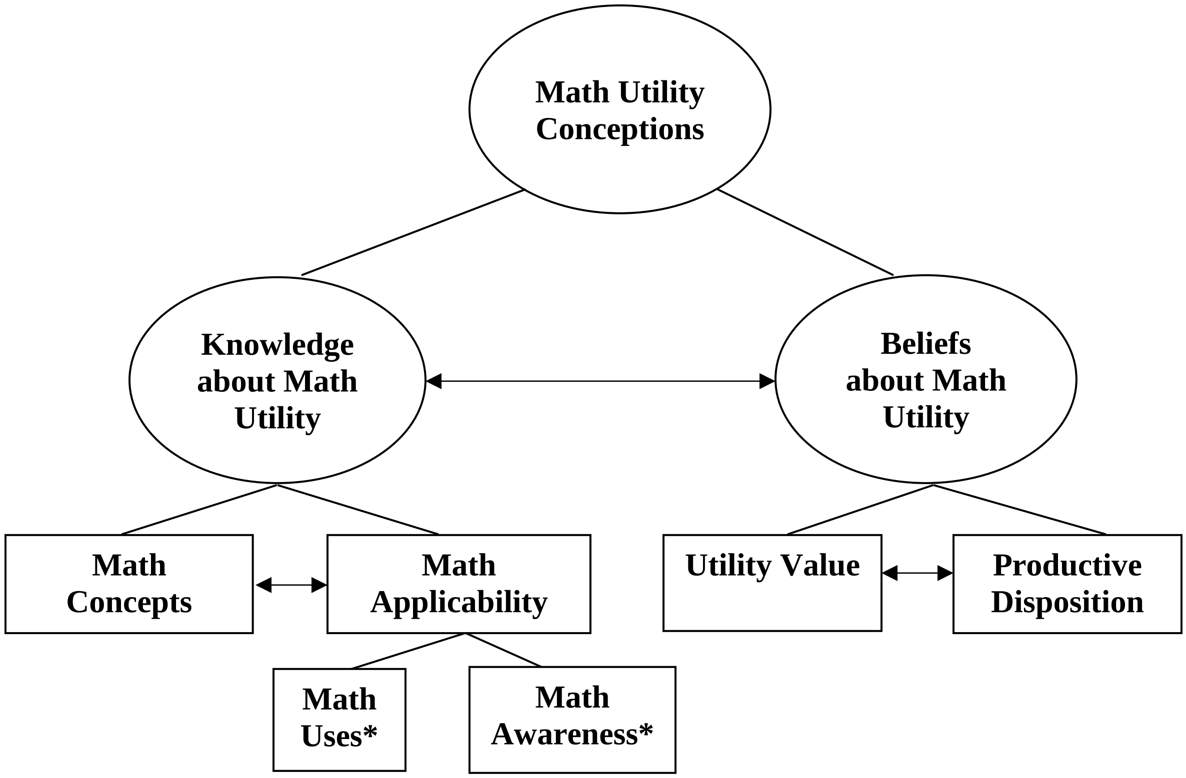

Figure 1. Math utility conceptions components model.
*Children only

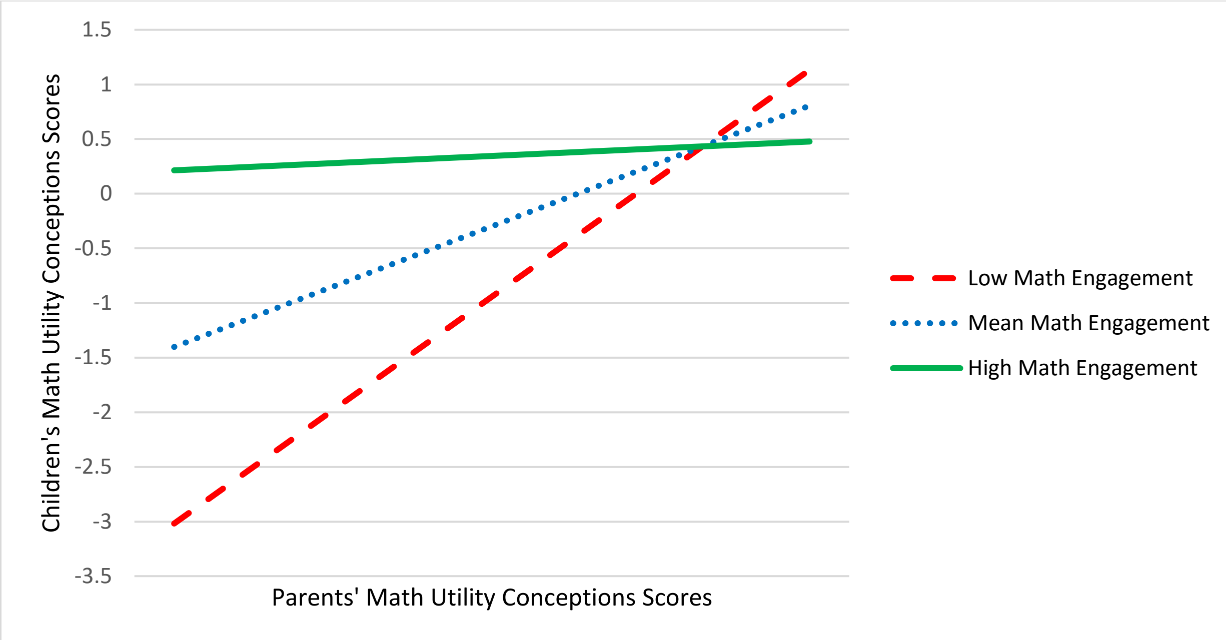

Figure 2. Children's math utility conceptions overall construct scores as a function of parents' math utility conceptions overall construct scores at low (-1 *SD*), mean, and high (+1 *SD*) parent-reported frequencies of children's math engagement.

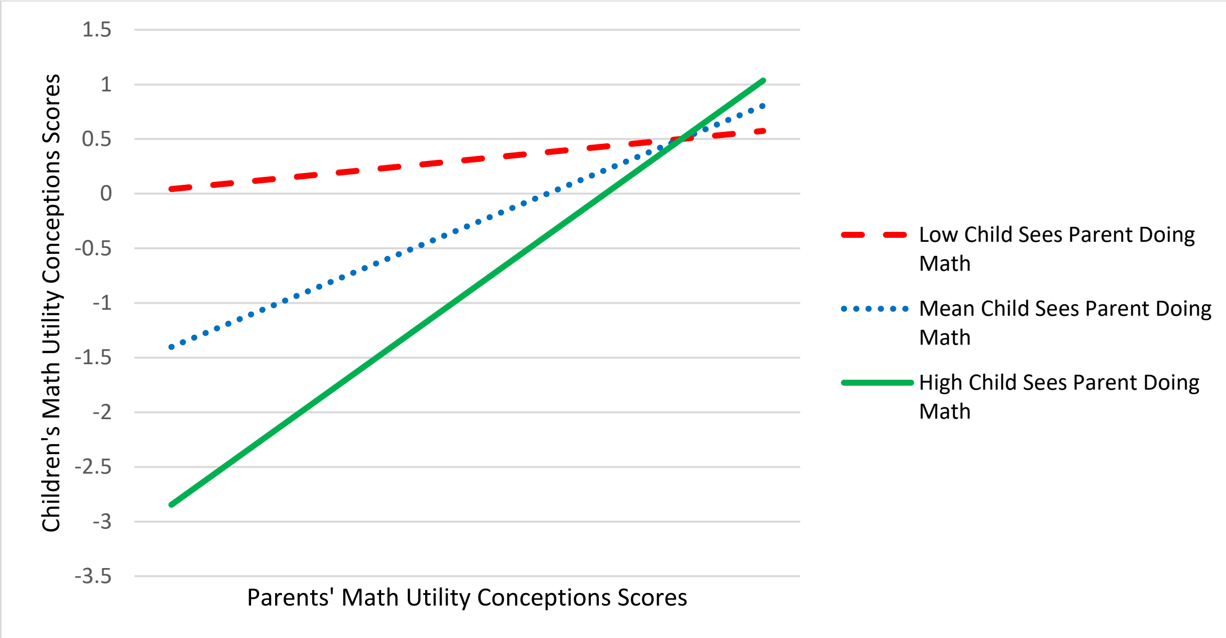

Figure 3. Children's math utility conceptions overall construct scores as a function of parents' math utility conceptions overall construct scores at low (-1 *SD*), mean, and high (+1 *SD*) frequencies of children seeing their parents doing math.

Chapter 4

Relations Between Elementary School Children's Conceptions about Math Utility and their Math Achievement

The National Assessment of Education Progress (NAEP) report shows that 60% of fourth grade children in the U.S. are not proficient in math and 20% of fourth graders score below basic levels of math performance (U.S. Department of Education, 2018). In order to improve proficiency in mathematics, it is important to determine the factors that contribute to mathematical learning. One such factor that is associated with mathematical learning is children's math conceptions (House, 2006; Muis, 2004; Mutodi, & Ngirande, 2014; National Research Council, 2001), which include children's knowledge, attitudes, and beliefs about math.

One important but understudied area of children's math conceptions is conceptions about the utility of math. Understanding how children's conceptions about the usefulness of math are related to their math achievement is important because early math skills predict later math achievement (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Duncan, et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; LeFevre, Polyzoi, Skwarchuk, Fast, & Sowinski, 2010). The limited research examining the relation between children's math utility conceptions and their math achievement shows a positive association (Mazzocco, Hanich, & Noeder, 2012; Rozek, Hyde, Svoboda, Hulleman, & Harackiewicz, 2015). However, these studies only considered one aspect of math utility conceptions (knowledge of usefulness of math and math utility value, respectively). To better understand children's math utility conceptions, it is important to examine two key constructs: children's *knowledge about math,* including math concepts and applicability of math, and *beliefs about math utility*, including utility value and productive disposition (for a visual representation of the conceptual model, see Figure 1).

Five Domains of Mathematics Development

Children begin learning mathematical principles, including conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (National Research Council, 2001) before they start preschool (Clements & Sarama, 2006; Ginsburg, Lee, & Boyd, 2008). Conceptual understanding is the comprehension of mathematical principles and operations that children typically learn in school. Procedural fluency is the ability to efficiently execute problem solving procedures with accuracy. Strategic competence is the ability to use various strategies to represent and solve math problems. Adaptive reasoning refers to the use of logical thought to explain and justify strategies and solutions to math problems. Unlike the first four domains, which refer to children's mathematical skills, productive disposition refers to children's beliefs about and attitudes towards math, specifically that math is useful and worthwhile. The National Research Council (2001) posits that these five domains are key, interrelated components of mathematical proficiency.

A large body of research over the last two decades has examined early mathematical development, including knowledge of number concepts (see Clements & Sarama, 2014; De Corte & Verschaffel, 2006; Geary, 2006, for reviews) and strategic approaches to solving mathematical problems (e.g., Siegler, 2003). Although research has explored the development of the skills identified in conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning, the fifth domain, productive disposition, remains largely understudied. The conceptual model used in this study, the *Math Utility Conceptions* model, extends the scope of how math utility conceptions are measured by including both *knowledge about math* and *beliefs about math* to examine the relations between children's math utility conceptions and math skills (For a more detailed description of the *Math Utility Conceptions* model, please see Study 2).

Conceptual understanding. The development of conceptual understanding in mathematics includes the development of early math concepts that are generally referred to in the literature as "number sense." Although many studies have explored the development of number sense (e.g., Baroody, 1987; Jordan, Kaplan, Oláh, & Locuniak, 2006; Odic, 2018; Siegler & Booth, 2004), researchers do not have one, shared definition. The National Council of Teachers of Mathematics (NCTM, 2000) identified the key Content Standards in which children develop math skills as number and operations, algebra, geometry, measurement, and data analysis and probability. Jordan et al. (2006) further broke down number and operations into counting, number knowledge, number transformation, estimation, and number patterns. Development of number sense begins as early as infancy (e.g., Starkey, Spelke, & Gelman, 1990; Xu & Spelke, 2000) and early number sense predicts mathematics achievement in elementary school (Duncan et al., 2007; Jordan, Glutting, & Ramineni, 2010; National Mathematics Advisory Panel, 2008).

Research with infants has shown that very young infants can distinguish between small quantities of objects and understand basic number transformations when researchers add or subtract an object from a set of objects (Geary, 2006; Jordan, Kaplan, Locuniak, & Ramineni, 2007). Using habituation studies, researchers have demonstrated that infants as young as five months old have an early understanding of simple mathematical concepts (see Ginsburg, Klein, & Starkey, 2000, for a review). Cooper (1984) found that infants between 12 and 18 months begin to be able to perform magnitude comparisons. They are able to discriminate between 1-, 2-, and 3-object sets and are able to identify when sets contain more or fewer objects than the previously presented set.

As children get older, they continue to build on these early skills and develop math concepts from their interactions with their environment (Clements & Sarama, 2006, 2014).

Children are able to learn early math concepts, or “everyday mathematics” principles, and are ready to build on them when they enter preschool (Ginsburg et al., 2008). This occurs because children begin to make use of available cultural tools, including number words and counting principles, to increase their knowledge of mathematical principles and math language, (Ginsburg et al., 2000). Early counting principles include one-to-one correspondence, when each number is assigned a specific name which is used when counting objects, the stable order principle, in which there is a consistent order in which objects are to be counted, and the cardinal principle, in which the final name of the number in a counting sequence reflects the number of objects in that sequence (Geary, 2006; Gelman & Gellistel, 1978; Jordan, et al, 2006).

Procedural fluency. In early elementary school, children further develop number sense and begin to develop procedural fluency, knowing when to use different math operations efficiently and accurately (National Research Council, 2001). Formal number transformation operations such as addition, subtraction, multiplication, division, fractions, and geometric reasoning are introduced as children enter formal schooling. Children build on the knowledge acquired prior to school and apply these numeric principles to increasingly complex problems. Counting principles are now used to learn simple addition and subtraction principles (Baroody, 1987). For example, children develop an understanding of the associative properties of addition [$(3+4)-2 = 3+(4-2)$] and the inverse principle of addition and subtraction (if $3+4=7$, then $7-4=3$, Geary, 2006). These properties are later applied to multiplication and division. Math curricula throughout elementary school also introduce concepts such as fractions, the base-10 system, estimation, algebra, and probability. Children have the ability to build upon their conceptual understanding of arithmetic principles as they continue formal schooling.

Strategic competence. As children progress in school, they learn multiple strategies for solving mathematics problems. Strategic competence is the ability to solve mathematical problems using various representations, or strategies (National Research Council, 2001). Acquiring these problem-solving strategies is closely linked to the development of procedural fluency, because knowing when and how to use various available procedures optimally leads to more efficient problem solving. The development of different strategies is important for the conceptual understanding necessary to solve more complex mathematics problems (Siegler, 2003).

Children learn and use new strategies as their capacity for working memory increases (Geary, 2006). For example, strategies for addition and subtraction begin with counting, with or without manipulatives. Children then begin to use their knowledge of addition principles to solve subtraction problems and vice versa. Later, children learn to use number patterns and count by 2s, 5s, or 10s to solve more efficiently addition and multiplication problems. As children gain better conceptual knowledge of number principles, they continue to learn new strategies for problem solving and can apply these strategies to more complex areas of mathematics learning.

Adaptive reasoning. The development of adaptive reasoning, the ability to think logically about the connections between mathematical principles and situations, generally does not begin until children are in secondary school (National Research Council, 2001). However, beginning reasoning skills can start earlier. In addition to the Content Standards identified by NCTM (2000), the Council also identified Process Standards of mathematical development that include problem solving, reasoning and proof, communication, connections, and representation. These processes expand beyond the math skills children are learning to ways the skills can be applied. For example, children may define math using the "connections" process, which refers to the notion that math can relate to other subjects, interests, and experiences. As children begin to better understand the connections that math has with other areas of their lives, they may begin to define math through processes as well as the content which they have studied.

Productive disposition. Productive disposition is composed of several beliefs about mathematics, including the beliefs that children are learners and doers of math, math is useful and relevant to children's daily lives, effort and engagement in mathematics will pay off, and math can be learned and used with effort (National Research Council, 2001). These beliefs are an integral part of developing the other four domains (conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning) of mathematics proficiency which researchers have long studied as areas of math development.

In addition to developing number sense, preschool age children are beginning to think mathematically and developing the ability to describe the ways in which they are learning. Metacognitive beliefs are children's beliefs about the nature of knowledge and how that knowledge is acquired (Muis, 2004). Metacognitive beliefs about math involve mathematical thinking and the ability to think about and express how they are solving simple problems (Ginsburg et al., 2008; Schoenfeld, 1992). For example, young children can describe how they would solve a problem using the knowledge and strategies they have acquired to reach a solution (Schoenfeld, 1989).

Although most research on children's metacognitive beliefs about math focuses on how children approach math problem solving (Muis, 2004), a small body of research has explored children's metacognitive beliefs about math utility. For example, Perlmutter, Bloom, Rose and Rogers (1997) examined kindergarten through third graders' math conceptions by using a semi-structured interview that included questions about what math is, who uses it, and how math is used at the grocery store and in cooking. Common responses to the question “What is math?” were related to numbers and mathematical operations, such as counting or addition/subtraction. When asked “What do people need to use math for?” children listed some “real life” uses for mathematics, including pressing the correct button in an elevator, counting out groceries in the grocery store, and paying bills. Mazzocco et al. (2012) asked second and third graders to define math and found that although they rarely defined math as useful, those who did had higher calculation skills. The present study will include productive disposition as an important aspect of math utility conceptions to better understand how it relates to children’s math achievement.

Math Utility Conceptions and Achievement

All five “strands” discussed above are important for the development of math proficiency (National Research Council, 2001), but there is little empirical evidence that examines how children’s conceptions of math utility relate to the other domains of math proficiency. Achievement motivation theories emphasize the relation between children’s beliefs, values, and goals and academic tasks (Wigfield et al., 2015; Wigfield, Eccles, Roeser, & Schiefele, 2008; Wigfield, Eccles, Schiefele, Roeser, & Davis-Kean, 2006), but math utility conceptions make up a very small portion of research examining these relations. Additionally, research examining these relations tends to focus only on math utility beliefs and overall math motivation beliefs, rather than both knowledge and beliefs about math utility. For this reason, the discussion of the current research will focus on research on beliefs.

Children’s beliefs about math positively relate to their math skills (e.g., Aunola, Leskinen, & Nurmi, 2006; De Corte & Verschaffel, 2006; Eccles, Wigfield, Harold, & Blumenfeld, 1993; Gottfried, Fleming, & Gottfried, 2001; Gottfried, Marcoulides, Gottfried, & Oliver, 2013; Gottfried, Marcoulides, Gottfried, Oliver, & Guerin, 2007; Mazzocco et al., 2012;

House, 2006; Murayama, Pekrun, Lichtenfeld, & vom Hofe, 2013). Research generally shows that a positive attitude toward learning mathematics will result in greater effort, higher self-efficacy in mathematics, and engagement in mathematical learning (e.g., De Corte & Verschaffel, 2006; Eccles et al., 1993; Muis, 2004; Wigfield et al., 2006). Three aspects of motivation beliefs that are particularly related to math achievement are self-concepts, performance expectations, and value of math education (e.g., Aunola et al., 2006; Marsh, Trautwein, Lüdtke, Köller, & Baumert, 2005; Mason, 2003; Muis, 2004; Pajares & Miller, 1994; Schoenfeld, 1989; Wigfield & Meece, 1988), all of which are included within the expectancy-value theory of motivation (Eccles & Wigfield, 2002; Eccles et al., 1993; Wigfield & Eccles, 2000).

Academic self-concepts refer to children's perceived competence in a given academic domain. Children develop self-concept beliefs through experiences with the environment and environmental reinforcements, like social comparison or teacher/parent feedback (Bong & Skaalvik, 2003). Children as young as preschool age demonstrate the ability to express distinctive competence perceptions across different academic domains, including math (Eccles, Wigfield, & Schiefele, 1998; Marsh, Ellis, & Craven, 2002). A large body of research shows that math self-concept beliefs are related to achievement (e.g., Green et al., 2012; Huang, 2011; Trautwein, Lüdtke, Köller, & Baumert, 2006), and several researchers argue that the relation between self-concept and achievement is reciprocal (Marsh & Martin, 2011; McInerney, Cheng, Mok, & Lam, 2012; Pinxten, Fraine, Damme, & D'Haenens, 2010; Seaton, Parker, Marsh, Craven, & Yeung, 2013). In other words, the feedback children receive about their math skills (e.g., test grades, verbal feedback, etc.) affects how they view their competency in math, which, in turn, impacts future achievement.

The value children place on math education refers to their interest in math and how useful they believe math is (math utility). Interest, or intrinsic value, is the enjoyment children get from participating in a specific activity or task; utility value is how well a task relates to current and future goals. Interest/intrinsic motivation theories posit that a person decides whether to engage in a task based on the significance of the task for them (e.g., Krapp, 2002; Renninger, Hidi, & Krapp, 1992; Ryan & Deci, 2000). Children who believe that mathematics is interesting, useful, and important are more likely to engage in mathematical activities and have higher math achievement (De Corte, Op't Eynde, & Verschaffel, 2002; Eccles & Wigfield, 2002; Eccles et al., 1993; Gottfried et al., 2001, 2007; Mason, 2003; Mazzacco et al., 2012; Murayama et al., 2013). In addition to math achievement, children who have higher utility value for math choose more rigorous math courses in high school (Mason, 2003; Simpkins & Davis-Kean, 2005; Simpkins, Davis-Kean, & Eccles, 2006).

As discussed in Metzger, Sonnenschein, and Galindo (2018, Study 1), Eccles and colleagues (e.g., Eccles et al., 1989, 1993; Wigfield et al., 1997) developed an expectancy-value theory that incorporates competence/expectancy and subjective task value (including interest and usefulness) beliefs, encompassing all three of the aforementioned aspects of motivation. Eccles and colleagues (e.g., Eccles, 2011; Eccles & Wigfield, 2002) examined the relation between expectancy-value beliefs for math and math achievement. The major strength of this model is the way that researchers bring together three very important aspects of math motivation: self-concepts, performance expectations, and task value. They and others have found that children's math motivations decrease as they progress through school (Dweck & Elliott, 1983; Eccles et al., 1993, 1998; Gottfried, Fleming, & Gottfried, 1998; Gottfried et al., 2001, 2007; Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002; King & McInerney, 2014; Nagy et al., 2010; Wigfield et al., 1997). Children’s subjective task value, in particular, tends to decline as children enter middle and high school. This suggests that older children’s interest in math and the extent to which they believe math is useful is generally lower than younger children.

Although the National Research Council (2001) highlights productive disposition as a factor important for developing math proficiency, little research has specifically examined the development of productive disposition and how these beliefs relate to math achievement for early elementary-age children. The limited research shows that children’s definitions that math is a useful tool in second grade were associated with their achievement in third grade, but that children did not often define math as being used outside the classroom (Mazzocco et al., 2012). Also, fourth grade children’s beliefs that success in math is due to working hard and studying are associated with higher math achievement (House, 2006). When they are asked whether specific activities featured math, children in higher elementary school grade levels were able to identify more real-life applications of math than children in lower grades (Metzger et al., 2018, Study 1). However, children’s awareness of the ways they use math in daily activities was still limited. It is possible that because the majority of children’s exposure to formal math is in the school context, it has shaped their views of math as a school-based activity. Children do not seem to connect the math they learn in school with the math they use in their everyday activities. The present study will examine this relation in more detail by measuring children’s math utility conceptions and their associated math skills.

Present Study

The primary goal of this study is to examine how children’s conceptions of math utility relate to children’s math achievement, assessed using four math subtests from the Woodcock-Johnson III Tests of Achievement (Woodcock, McGrew, & Mather, 2001), to understand how these skills relate to children's math utility conceptions. We hypothesize that the higher their overall math utility conceptions scores, the higher their math achievement. In other words, the more knowledge children have about the utility of math and the more strongly they endorse beliefs that math is useful, the higher their achievement levels.

Another goal of this study is to explore the relations between specific aspects of math utility conceptions and children's math achievement. Although this research aim is primarily exploratory, we hypothesize that productive disposition will significantly predict math achievement (National Research Council, 2001). We also hypothesize that math applicability will be more likely to relate specifically to math reasoning skills, because math reasoning measures the application of math concepts in more "real-world" problems.

Another goal of this study is to explore whether there are grade-related differences in how children's overall math utility conceptions as well as specific aspects of math utility conceptions relate to their math achievement. Although math skills and utility conceptions change as children get older, there is not enough prior evidence to hypothesize whether there are grade-related differences in how they are associated with each other.

This study adds to the literature by: 1) exploring the relation between an expanded model of math utility conceptions and children's math achievement, 2) highlighting which aspects of math utility conceptions may be more predictive of children's math achievement, and 3) examining whether the relation between math utility conceptions and math achievement differs across grade groups.

Method

Participants

One hundred four children (55% girls) were recruited during the summer and early fall of 2017 from schools and summer camps in the Baltimore/Washington, D.C. corridor. Most of the children (88%) were interviewed during the summer. Twenty-eight children (Mean age = 6.60 years, *SD* = 0.41) were entering or had just entered first grade, 26 second grade (Mean age = 7.36 years, *SD* = 0.31), 26 third grade (Mean age = 8.29 years, *SD* = 0.35), and 24 fourth grade (Mean age = 9.41 years, *SD* = 0.36). For grade comparisons, children were grouped into younger (rising first and second graders, *n* = 54) and older (rising third and fourth, *n* = 50) grade groups (Simons, Metzger, & Sonnenschein, 2018; Sobel & Letourneau, 2015). Children came from diverse racial/ethnic backgrounds. Fifty-seven percent of child participants were European American/White (*n* = 59), 15% African American/Black (*n* = 16), 14% Asian/Pacific Islander (*n* = 14), 3% Hispanic/Latino (*n* = 3), 3% another race/ethnicity (*n* = 3), and 9% multiracial (*n* = 9).

The majority of child participants had highly-educated parents. Fifty-seven percent (*n* = 59) of children had parents who earned a post-graduate degree, 28% (*n* = 29) Bachelor's degree, and 15% (*n* = 16) some college or an Associate's degree). The median total household income for this sample was over $125,000. Fifty-four percent (*n* = 56) of children came from household with a combined income of $125,000 or more, 32% (*n* = 33) $75,000 - $125,000, and 10% (*n* = 10) less than $75,000.

Measures

A complete version of the *Children's Mathematics Conceptions Questionnaire* can be found in Appendix A. Below is a description of specific measures within the questionnaires for each aspect of the math utility conceptions model and children's home math engagement. These descriptions were also found in Chapter 3 (Study 2).

Knowledge about math.

Math concepts. Children were asked "What is math?" To measure the extent to which children define math as a useful tool, a scale based on Mazzocco et al. (2012) was developed. This five-point scale ranged from a 0, ("*I cannot explain*") to 2 (*"it has to do with numbers"*) to 4 ("*Math is an important thing you use every day, like at the grocery store*"). The score represents the *math concepts* score within the overall math utility composite.

Inter-rater reliability for this and other scales was established by having two raters independently code about 25% of the responses. Because the data are on a continuous scale, an intraclass correlation coefficient (ICC, McGraw & Wong, 1996) was used to determine acceptable inter-rater reliability. Specifically, two-way random-effects model ICCs with absolute agreement (Koo & Li, 2016) were used. If acceptable ICC values of .75 or higher, representing good reliability (Koo & Li, 2016), were not reached in the first round of coding, a new set of responses were coded. This process continued until all ICC values were at least .75. Remaining responses were coded by one of the raters. The final ICC for the math concepts scale was .84.

Applicability of math. Two open-ended questions were used to examine this construct. Children were asked, "Who uses math?" and "How does {person[s] mentioned} use math?" Similar to math concepts, a scale was developed based on children's responses to these questions to represent the extent of children's knowledge about how math is useful in daily life. The five-point scale ranged from 0, ("*I cannot think of anyone*") to 2 (*"everyone because they have to learn it in school"*) to 4 (*"Everyone has to use math in their life all the time- like at the grocery* *store"*). This scale score represented a *math uses* score within the math applicability composite. The final ICC for the math uses scale was .80.

To measure awareness of math in daily activities, children were asked whether and how math can be used in different activities. There were 10 activities: playing board games, card games, and video games, doing puzzles, cooking, helping at the grocery store, building with blocks or Legos, using or playing with money, using maps or a globe, and keeping score in games or sports. Children were asked, "Some children think math is used when they play board games, some think math is not used at all. Do you think math is used when you play board games?" If children responded "yes," they were asked how math was used in the activity. As in Metzger et al. (2018, Study 1), children's responses were coded on a 4-point scale: 0 if the child did not identify that math was used or if s/he said that math was used, but the description was not related to math; 1 if the child said that math was used in the activity, but did not elaborate about how; 2 if s/he described a basic math skill; 3 if s/he described an advanced math skill. The final ICCs for children's math awareness items ranged from .76 to .98. A composite was created by averaging scores for all activities. This composite scale score represented the *math awareness* score within the math applicability composite. Cronbach's alpha for the math awareness scale was .82.

Finally, a composite was created by summing the *math uses* and *math awareness* scale scores. This scale score represented the *math applicability* score within the overall math utility composite.

Beliefs about math.

Utility value. To measure utility values, children were asked nine five-point scale items to create a utility value scale score. Items were adapted from similar measures used to measure mathematics or reading motivation (Baker & Scher, 2002; Eccles et al., 1993; Sonnenschein, Baker, & Garrett, 2004; Wigfield & Guthrie, 1997). Examples included "Math is useful for everyone" and "It is important for me to learn math." Children were asked to report whether they felt each item is 1 "not at all like me," to 5 "a lot like me." Three non-math activities were presented as examples at the outset in order to familiarize children with the rating scale. A composite was created by averaging the scores on the nine items. This composite scale represented *utility value* score within the overall math utility composite. Cronbach's alpha for the utility value scale was .79.

Productive disposition. To measure children's dispositions towards math, they were asked 10 five-point scale items. Similar to the utility value scale, some items were adapted from similar measures used to measure mathematics or reading motivation (Baker & Scher, 2002; Eccles et al., 1993; Sonnenschein et al., 2004; Wigfield & Guthrie, 1997). Examples included "Working hard in math helps me do better in math" and "Learning math is worth my time." Children were asked to report whether they felt each item is 1 "not at all like me," to 5 "a lot like me." A composite was created by averaging the scores on all items. This composite scale represented *productive disposition* score within the overall math utility composite. Cronbach's alpha for the productive disposition scale was .87.

Overall math utility conceptions. To create an overall math utility conceptions scale score, the scale scores for each aspect of the math utility conceptions model (math concepts, math applicability, math utility, and productive disposition) were standardized, then summed. This allowed each aspect to be weighted equally and be on a standardized scale. The higher this composite score, the farther someone is above the mean math utility conception score for this sample.

Math skills. To assess children's math skills, children were tested using four subtests from the Woodcock Johnson-III *Tests of Achievement* (Woodcock et al., 2001): Calculation, Math Fluency, Applied Problems, and Quantitative Concepts (which includes Concepts and Number Series). Median reliability for Calculation was .85 in the five- to 19-year age range, Math Fluency is .97 in the seven- to 19-year age range, Applied Problems is .92 in the five- to 19-year age range, and Quantitative Concepts is .90 in the five- to 19-year age range (McGrew, Schrank & Woodcock, 2007). Reliabilities and standardized scores on these measures are based on the Woodcock-Johnson III Normative Update (WJ-III NU, McGrew et al., 2007; Woodcock, McGrew, Schrank, & Mather, 2001, 2007).

For analyses, overall math achievement was measured using the Broad Math cluster and related subtests were combined into two additional cluster scores: Math Calculations cluster and the Math Reasoning cluster. The Broad Math cluster assesses problem solving, number facility, automaticity, and reasoning with the Calculation, Math Fluency, and Applied Problems subtests. This cluster has a median reliability of .95 in the five- to 19-year age range (Mather & Woodcock, 2001; McGrew et al., 2007). The Math Calculations cluster assesses skills in performing written calculations with the Calculation and Math Fluency subtests. This cluster has a median reliability of .91 in the five- to 19-year age range. The Math Reasoning cluster assesses problem solving, number facility, automaticity, and reasoning with the Applied Problems and Quantitative Concepts subtests. This cluster has a median reliability of .95 in the five- to 19-year age range. To control for grade-level differences in math achievement, standard scores using grade-based norms for the three subtest clusters were used in analyses.

Demographic information. As part of the consent documents, parents provided their child's age, grade in school in the fall, gender, and race/ethnicity (African American/Black, European American/White, Hispanic/Latino, Asian/Pacific Islander, or "other"). They also provided their own age, race/ethnicity, relation to the child (mother, father, or "other"), highest level of education they received (post-graduate degree, Bachelor's degree, Associate's degree, some college/vocational/technical, high school graduate, or less than high school), and total household income ($125,000 or more, $100,000 - $124,999, $75,000 - $99,999, $50,000 - $74,999, $25,000 – $49,999, or less than $25,000).

Procedure

Children were interviewed and tested individually by a trained graduate student or advanced undergraduate research assistant. Each session took place in a research lab, the child's home, or a public library. Researchers administered the *Children's Mathematics Conceptions Questionnaire* first, then the Woodcock Johnson-III *Tests of Achievement* Calculation, Math Fluency, Applied Problems, and Quantitative Concepts subtests. Interviews were recorded using a digital recorder and the interviewer also took notes of the child's responses. Interviews lasted about 15-20 minutes, and testing sessions lasted about 30-45 minutes. All sessions were conducted in English.

Results

Preliminary Analyses

Preliminary analyses were conducted to determine which demographic variables, if any, should be included as covariates in analyses. First, independent-samples t-tests were used to determine whether there were gender differences in the math achievement clusters. There were no significant gender differences on any of the math cluster standard scores, $p > .05$. Then, using a one-factor ANOVA, we examined whether there were differences between the race/ethnicity groups. There were no significant race/ethnicity differences on any of the math cluster standard scores, $p > .05$. Because of the lack of significant gender and race/ethnicity differences, neither variable was used as a covariate in analyses.

We also examined whether socioeconomic factors, parents' highest education earned and household income, were correlated with children's math achievement using Spearman's rho correlations. Parents' highest education level was significantly correlated with children's Broad Math cluster scores, $r(102) = .25$, $p = .009$, Math Calculations cluster scores, $r(102) = .30$, $p = .002$, and Math Reasoning cluster scores, $r(102) = .21$, $p = .033$. Household income was significantly correlated with children's Broad Math cluster scores, $r(97) = .30$, $p = .002$, Math Calculations cluster scores, $r(97) = .26$, $p = .010$, and Math Reasoning cluster scores, $r(97) = .27$, $p = .006$. Because socioeconomic factors were significantly correlated with many of the math achievement outcomes, and weakly correlated with each other, $r(97) = .23$, $p = .023$, both parents' highest education and household income were included as covariates in the analyses.

Children's Math Utility Conceptions

Detailed descriptions of children's math utility conceptions can be found in Study 2.

Math utility knowledge. The mean math concepts scale score was 2.11 ($SD = 0.75$) on a 0-4 scale. Most children (78%) received a score of two on the math concepts scale, which reflects knowledge of math focused primarily on numbers and operations. Only 14% of children provided responses that indicated that they defined math as something that is connected to other subjects or daily life activities (3 or 4 on the scale). The mean math uses scale score was 2.28 ($SD = 1.24$) on a 0-4 scale. About 26% scored a four, demonstrating that about one-fourth of the sample indicated and described how everyone uses math in their daily lives. When children were asked to describe how math was used in 10 different activities, the mean math awareness composite score was 1.48 ($SD = 0.68$) on a 0-3 scale, which corresponds with children being able to indicate that math is included in an activity and sometimes being able to describe that it uses a basic math operation. The math uses and math awareness scale scores were summed into the math applicability scale score, which had a mean of 3.75 (*SD* = 1.65).

Math utility beliefs. The mean math utility composite score was 4.22 (*SD* = 0.67) on a 1-5 scale, indicating that, on average, children see math as useful and important. The mean productive composite score was 4.15 (*SD* = 0.79) on a 1-5 scale, indicating that, on average, children see themselves as math users and that working hard in math is worthwhile.

Overall math utility conceptions composite. The overall math utility conceptions scale score was the sum of the math concepts, math applicability, math utility, and productive disposition standardized scores, which had a mean of -0.09 (*SD* = 2.48).

Children's Math Achievement

Children's average cluster standard scores generally were in the average or high average range (see Table 1). Paired-samples t-tests showed that children's math reasoning standards scores were significantly greater than their math calculation scores (see Table 2). These differences were consistent whether we examined the cluster or subtest scores.

Relations Between Math Utility Conceptions and Math Achievement

To assess the relations between children's overall math utility conceptions and their math achievement, three separate OLS regressions were used with children's overall math utility conceptions as the predictor, parents' education and income as covariates, and each of the math achievement cluster standard scores as the outcomes. Children's math utility conceptions significantly predicted their Math Reasoning cluster scores, $\beta = .215$, $t(95) = 2.16$, $p = .034$. However, children's math utility conceptions did not significantly predict the Broad Math, $\beta = .155$, $t(95) = 1.58$, $p = .118$, or Math Calculations cluster scores, $\beta = .104$, $t(95) = 1.06$, $p = .292$.

To determine whether the relations between children's math utility conceptions and achievement were different for younger and older children, three separate OLS regressions were used with children's overall math utility conceptions, grade group, and the interaction between conceptions and grade group as predictors, parents' education and income as covariates, and each of the math achievement cluster standard scores as the outcomes. Grade group was not a significant moderator between children's math utility conceptions and their Broad Math, β = .388, $t(93) = 1.19$, $p = .235$, Math Calculation, β = .361, $t(93) = 1.12$, $p = .265$, or Math Reasoning cluster scores, β = .472, $t(93) = 1.43$, $p = .157$.

We further explored the relations between math utility conceptions and math achievement by examining whether specific aspects of math utility predicted cluster scores. We used three separate OLS regressions with children's math concepts, math applicability, math utility, and productive disposition scale scores as predictors, parents' education and income as covariates, and each of the math achievement cluster standard scores as the outcome. For each of the clusters, children's productive disposition was the only significant predictor of children's math achievement (see Table 3).

Finally, we explored whether the relations between individual aspects of math utility conceptions and achievement were different for younger and older children. We used three separate OLS regressions with each of the four aspects of utility conceptions, grade group, and four interaction terms between each aspect of conceptions and grade group as predictors, parents' education and income as covariates, and each of the math achievement cluster standard scores as the outcomes (Table 4). We probed significant interactions with grade group using a simple slopes analysis (Cohen, Cohen, West & Aiken, 2003; Hayes & Rockwood, 2017; Preacher, Curran, & Bauer, 2006). For each outcome, we estimated the relation between the relevant math utility conception score and the math achievement score for younger and older children, holding all other variables constant at their respective mean values.

For Broad Math cluster scores, there were significant interactions between grade group and math applicability, $p = .005$, and grade group and math utility, $p = .016$. The relation between math applicability and Broad Math scores is positive for younger children and negative for older children (see Figure 2). Conversely, the relation between math utility and Broad Math is negative for younger children and positive for older children (see Figure 3).

For Math Calculations cluster scores, there was a significant interaction between grade group and math applicability, $p = .029$. Similar to Broad Math, the relation between math applicability and Math Calculations scores is positive for younger children and negative for older children (see Figure 4).

For Math Reasoning cluster scores, there was a significant interaction between grade group and math utility, $p = .033$. Similar to Broad Math scores, the relation between math utility and Math Reasoning is negative for younger children and positive for older children (see Figure 5).

Discussion

This study explored the relations between elementary-age children's math utility conceptions and children's math skills. Researchers who have investigated this relation in the past have done so with limited measures of math utility and primarily with older children (Guo, Marsh, Parker, Morin, & Yeung, 2015; Marsh & Martin, 2011; Mazzocco et al., 2012; Musu-Gillette, Wigfield, Harring, & Eccles, 2015). Contrary to our hypothesis, overall math utility conceptions did not predict math achievement. Instead, productive disposition was the only aspect of math utility conceptions that was a significant predictor of all math skill clusters. This finding supports the National Research Council's (2001) five strand model in which productive disposition is intertwined with several areas of math skills development. This effect may also be bidirectional. In other words, it may be that if children find math useful and worthwhile they work harder to succeed in math. It also may be that children who do well in math see the value of their hard work because it has already paid off.

We also explored whether there were differences in the relations between math utility conceptions and math achievement for different grade groups, and the results of this study suggest a complex relation between math utility conceptions and math achievement. Grade group did not moderate the relation between overall math utility conceptions and math achievement. However, when we explored whether grade group moderated the relations between specific aspects of math utility conceptions and math achievement, we found differences in the relations between math applicability and math utility and children's math achievement for older and younger children.

For younger children (first and second graders), higher math skills were associated with higher math applicability scores, which included the extent to which they could identify how people use math in their lives and how math was used in specific activities. For older children (third and fourth graders), however, this relation was negative. To better understand why there is a negative relation between math applicability and math achievement for older children, we examined children's scores on the components of math applicability, the math uses and the math awareness scale scores. We found that, for older children, children who scored a two (everyone uses math, because they learn it in school) on the *math uses* scale were associated with higher math achievement scores than children who scored a four (everyone uses math in their daily lives). One explanation for this is that perhaps older children who are more proficient in math may also think of math as more school-based in terms of the context in which people use math, which would have given them lower scores on this scale. Additionally, it could be that older children may have developed the knowledge of how people use math in their daily lives, even though they are not as proficient at math themselves. In other words, even when children have lower math skills, they may still possess the knowledge that people use math in daily activities.

Additionally, for older children, higher math skills were associated with higher math utility. However, for younger children, this relation was negative. Perhaps as children get older and learn more about the utility value of math, this belief drives their motivation to succeed in math more so than when they are younger. Again, this may be a bidirectional effect, such that older children who see math as useful may work harder in math and, in turn, develop greater proficiency, and/or older children who are successful in math see it as inherently more useful and worthwhile, but this effect is not the same for younger children who may not have learned as much about how the math they learn in school can be tied to math in their everyday life.

Implications for Practice

Improving children’s math skills is an educational priority (National Mathematics Advisory Panel, 2008); exploring children’s understanding of math and its uses provides insight into the development of children’s math proficiency. One way to improve math proficiency may be to include more real-world applications of math into elementary math curricula. Currently, the extent to which this is done is primarily by using word problems with “real” math examples, but it may be necessary for teachers to find more innovative ways to connect “school math” with ways that math is used in daily life activities. For example, when teaching fractions, teachers could use a measurement activity in which children must measure furniture or pictures and determine where they could fit in a room. Learning to use fractions in this way would use fraction problems that children and adults actually use in their lives (e.g., 1/4 + 5/8), rather than fractions that they are unlikely to ever encounter (e.g., 6/19 + 5/7) and could lead to more explicit discussion of how the process of adding fractions is relevant and important to learn. An example of a related assignment would be to have a math journal activity where children are instructed to find ways in which math is used outside of school and present their findings in class. Activities like this may help make connections more salient for children and improve their knowledge and beliefs about math utility. After explicitly talking about these connections, children may be better able to articulate more real-life applications of math.

Limitations and Future Directions

Although this study provides more support for the link between children's math utility conceptions and their math achievement, there are some key limitations to consider. The sample is a relatively small convenience sample and findings may not be generalizable to all demographic groups. We know that parent's highest education level and family income were significantly related to children's achievement, but we were unable to determine whether there were any important differences in children's math conceptions between children from families of differing socioeconomic status. Additionally, child responses may have been limited by children's verbal abilities and may not capture their complete math utility conceptions. That being said, scales were developed based on content of responses, rather than linguistic sophistication of response to limit the effect of developmental differences. For example, two children received the same score for the following answers: "at the grocery store, they give you change back" and "the cashier has to count how much money you gave them and how much owe and calculate how much change to give you back." Research shows that children as young as preschool-age are able to respond effectively to both open- and closed-ended questions about math self-concepts and home math engagement (Eccles et al., 1993; Marsh et al., 2002; Mazzocco et al., 2012; Ramani & Siegler, 2008; Wigfield et al., 1997). Despite these limitations, this study offers important information to increase our understanding of how children's math utility conceptions are related to children's math achievement.

Findings from this study could provide the basis for continued study of math utility conceptions, which will lead to a better understanding of how we can incorporate these conceptions into math curricula for elementary-age children. Some school-based intervention research has found that productive dispositions towards math can be improved (Graven, 2015; Jansen, 2012; Mitchell, 1999). Jansen (2012) examined which teaching styles during small group work in two sixth-grade classrooms increased children's productive dispositions towards math. She found that when teachers provided children with more autonomy, sought multiple solutions to problems, and encouraged conceptual understanding of the material, children had more productive dispositions than with teachers who did not employ these strategies. Similarly, Graven (2015) found that extracurricular math clubs for struggling children where teachers could engage with small groups of third and fourth graders provided an environment in which children could improve their dispositions towards math. Mitchell (1999) found that using a weekly reward system for performance (effort, achievement, and attitude) in math with six- to eight-year-olds was associated with increased positive attitudes towards math and effort during math work. These interventions suggest that altering instructional practices can be used to increase children's productive disposition, which is related to their math achievement. Further research should specifically evaluate how increase in math utility conceptions corresponds with increases in math proficiency.

Conclusion

The goal of this study was to examine the relation between children's math utility conceptions and their math achievement. This study provides evidence that children's knowledge and beliefs about the usefulness of math play a role in their math achievement, but this relation is complex. For all children, results suggest that a productive disposition towards math is associated with children's math achievement. However, for other aspects of knowledge and beliefs about math utility, namely math uses and math utility value, the relations between math utility conceptions and achievement is different for younger and older elementary school-aged children. These results may guide math curriculum development for elementary-aged children to more intentionally use real-world applications to teach math concepts and, in doing so, improve children's understanding of the importance of math in their daily lives.

References

Aunola, K., Lesiken, E., Lerkkanen, M. K., & Nurmi, J. E. (2004). Developmental dynamics of math performance from preschool to Grade 2. *Journal of Educational Psychology, 96*, 699-713. doi:10.1037/0022-0663.96.4.699

Aunola, K., Leskinen, E., & Nurmi, J.-E. (2006). Developmental dynamics between mathematical performance, task motivation, and teachers' goals during transition to primary school. *British Journal of Educational Psychology, 76*, 21-40. doi:10.1348/000709905X51608

Baker, L., & Scher, D. (2002). Beginning reader's motivation for reading in relation to parental beliefs and home reading experiences. *Reading Psychology, 23*, 239-269. doi:10.1080/713775283

Baroody, A. J. (1987). The development of counting strategies for single-digit addition. *Journal for Research in Mathematics Education, 18*, 141 – 157. doi:10.2307/749248

Bong, M., & Skaalvik, E. M. (2003). Academic self-concept and self-efficacy: How different are they really? *Educational Psychology Review, 15*, 1-40. doi:10.1023/A:1021302408382

Clements, D. H., & Sarama, J. (2006). Your child's mathematical mind. *Scholastic Parent & Child, 14*, 30-39.

Clements, D. H., & Sarama, J. (2014). *Learning and teaching early math: The learning trajectories approach (2nd ed.).* New York: Routledge.

Cohen, J., Cohen, P., West, S. G., & Aiken, L. S. (2013). *Applied multiple regression/correlation analysis for the behavioral sciences*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Cooper, R.G. (1984). Early number development: Discovering number space with addition and subtraction. In C. Sophian (Ed.), Origins of cognitive skills (pp. 157-192). Hillsdale, NJ: Erlbaum.

De Corte, E., Op't Eynde, P., & Verschaffel, L. (2002). "Knowing what to believe": The relevance of students' mathematical beliefs for mathematics education. In B. K. Hofer & P. R. Pintrich (Eds.), *Personal epistemology: The psychology of beliefs about knowledge and knowing* (pp.297-320). Mahwah, NJ: Lawrence Erlbaum Associates.

De Corte, E. & Verschaffel, L. (2006). Mathematical thinking and learning. In W. Damon & R. M. Lerner (Series Eds.), & K. A. Renninger, & I. E. Sigel (Vol. Eds.), *Handbook of Child Psychology: Vol. 4 Child psychology in practice* (6^{th} ed., pp. 103-152). New York: Wiley.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., … Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, *43*, 1428-1446. doi:10.1037/0012-1649.43.6.1428.supp

Dweck, C. S., & Elliott, E. S. (1983). Achievement motivation. In E. M. Hetherington (Ed.), *Socialization, personality, and social development.* New York: Wiley.

Eccles, J. (2011). Gendered educational and occupational choices: Applying the Eccles et al. model of achievement-related choices. *International Journal of Behavioral Development*, *35*, 195-201. doi:10.1177/0165025411398185

Eccles, J. S., & Wigfield, A. (2002). Motivational beliefs, values, and goals. *Annual Review of Psychology*, *53*, 109-132. doi:10.1146/annurev.psych.53.100901.135153

Eccles, J. S., Wigfield, A., Flanagan, C. A., Miller, C., Reuman, D. A., & Yee, D. (1989). Self-concepts, domain values, and self-esteem: Relations and changes at early adolescence. *Journal of Personality*, *57*, 283-310. doi:10.1111/j.1467-6494.1989.tb00484.x

Eccles, J. S., Wigfield, A., Harold, R., & Blumenfeld, P. (1993). Age and gender differences in children's self-and task perceptions during elementary school. *Child Development, 64*, 830-847. doi:10.1111/j.1467-8624.1993.tb02946.x

Eccles, J. S., Wigfield, A., and Schiefele, U. (1998). Motivation to succeed. In W. Damon (Series ed.) and N. Eisenberg, *Handbook of Child Psychology. Vol. 3: Social, Emotional, and Personality Development* (5th Ed., pp. 1017-1095). New York: Wiley.

Geary, D. C. (2006). Development of mathematical understanding. In W. Damon & R. M. Lerner (Series Eds.) & D. Kuhn & R. Siegler (Vol. Eds.), *Handbook of child psychology: Vol. 2 Cognition, perception, and language* (6th ed., pp. 777-810). New York: Wiley.

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number.* Cambridge, MA: Harvard University Press.

Ginsburg, H. P., Klein, A., Starkey, P. (2000). The development of children's mathematical thinking: Connecting research with practice. In W. Damon (Series Ed.) & I. E. Sigel & K. A. Renninger (Vol. Eds.), *Handbook of child psychology: Vol. 4 Child Psychology in Practice* (5th ed., pp. 401-476). New York: Wiley.

Ginsburg, H. P., Lee, J. S., Boyd, J. S. (2008). Mathematics education for young children: What it is and how to promote it. *Social Policy Report, 22*, 3-22.

Gottfried, A. E., Fleming, J. S., & Gottfried, A. W. (1998). Role of cognitively stimulating home environment in children's academic intrinsic motivation: A longitudinal study. *Child Development*, *69*, 1448-1460. doi:10.2307/1132277

Gottfried, A. E., Fleming, J. S., & Gottfried, A. W. (2001). Continuity of academic intrinsic motivation from childhood through late adolescence: A longitudinal study. *Journal of Educational Psychology, 93*, 3-13. doi:10.1037//0022-0663.93.1.3

Gottfried, A. E., Marcoulides, G. A., Gottfried, A. W., & Oliver, P. H. (2013). Longitudinal pathways from math intrinsic motivation and achievement to math course accomplishments and educational attainment. *Journal of Research on Educational Effectiveness, 6*, 68-92. doi:10.1080/19345747.2012.698376

Gottfried, A. E., Marcoulides, G. A., Gottfried, A. W., Oliver, P. H., & Guerin, D. W. (2007). Multivariate latent change modeling of developmental decline in academic intrinsic math motivation and achievement: Childhood through adolescence. *International Journal of Behavioral Development*, *31*, 317-327. doi:10.1177/0165025407077752

Graven, M. (2015). Strengthening maths learning dispositions through ‘math clubs.’ *South African Journal of Childhood Education*, *5*, 1-7. doi:10.4102/sajce.v5i3.342

Green, J., Liem, G. A. D., Martin, A. J., Colmar, S., Marsh, H. W., & McInerney, D. (2012). Academic motivation, self-concept, engagement, and performance in high school: Key processes from a longitudinal perspective. *Journal of Adolescence*, *35*, 1111-1122. doi:10.1016/j.adolescence.2012.02.016

Guo, J., Marsh, H. W., Parker, P. D., Morin, A. J., & Yeung, A. S. (2015). Expectancy-value in mathematics, gender and socioeconomic background as predictors of achievement and aspirations: A multi-cohort study. *Learning and Individual Differences*, *37*, 161-168. doi:10.1016/j.lindif.2015.01.008

Hayes, A. F., & Rockwood, N. J. (2017). Regression-based statistical mediation and moderation analysis in clinical research: Observations, recommendations, and implementation. *Behaviour Research and Therapy*, *98*, 39-57. doi:10.1016/j.brat.2016.11.001

House, J. D. (2006). Mathematics beliefs and achievement of elementary school students in Japan and the United States: Results from the Third International Mathematics and Science Study. *The Journal of Genetic Psychology*, *167*, 31-45.

Huang, C. (2011). Self-concept and academic achievement: A meta-analysis of longitudinal relations. *Journal of School Psychology*, *49*, 505-528. doi:10.1016/j.jsp.2011.07.001

Jacobs, J. E., Lanza, S., Osgood, D. W., Eccles, J. S., & Wigfield, A. (2002). Changes in children's self-competence and values: Gender and domain differences across grades one through twelve. *Child Development*, *73*, 509-527. doi:10.1111/1467-8624.00421

Jansen, A. (2012). Developing productive dispositions during small-group work in two sixth-grade mathematics classrooms: Teachers' facilitation effort and students' self-reported benefits. *Middle Grades Research Journal*, *7*, 37-56.

Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences*, *20*, 82-88. doi:10.1016/j.lindif.2009.07.004

Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research and Practice*, *22*, 36-46. doi:10.1111/j.1540-5826.2007.00229.x

Jordan, N. C., Kaplan, D., Oláh, L. N., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child Development*, *77*, 153-175. doi:10.1111/j.1467-8624.2006.00862.x

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, *45*, 850-867. doi:10.1037/a0014939

King, R. B., & McInerney, D. M. (2014). Mapping changes in students' English and math self-concepts: a latent growth model study. *Educational Psychology*, 17 pages. doi:10.1080/01443410.2014.909009.

Koo, T. K., & Li, M. Y. (2016). A guideline of selecting and reporting intraclass correlation coefficients for reliability research. *Journal of Chiropractic Medicine, 2016,* doi:10.1016/j.jcm.2016.02.012.

Krapp, A. (2002). Structural and dynamic aspects of interest development: Theoretical considerations from an ontogenetic perspective. *Learning and Instruction*, *12,* 383-409. doi:10.1016/S0959-4752(01)00011-1

LeFevre, J.-A., Polyzoi, E., Skwarchuk, S.-L., Fast, L., & Sowinski, C. (2010). Do home numeracy and literacy practices of Greek and Canadian parents predict the numeracy skills of kindergarten children? *International Journal of Early Years Education, 18*, 55-70. doi:10.1080/09669761003693926

Marsh, H. W., Ellis, L. A., & Craven, R. G. (2002). How do preschool children feel about themselves? Unraveling measurement and multidimensional self-concept structure. *Developmental Psychology, 38*, 376-393. doi:10.1037//0012-1649.38.3.376

Marsh, H. W., & Martin, A. J. (2011). Academic self-concept and academic achievement: Relations and causal ordering. *British Journal of Educational Psychology*, *81*, 59-77. doi:10.1348/000709910X503501

Marsh, H. W., Trautwein, U., Lüdtke, O., Köller, O., & Baumert, J. (2005). Academic self-concept, interest, grades, and standardized test scores: Reciprocal effects models of causal ordering. *Child Development, 76*, 397-416. doi:10.1111/j.1467-8624.2005.00853.x

Mason, L. (2003). High school students' beliefs about maths, mathematical problem solving, and their achievement in maths: A cross-sectional study. *Educational Psychology, 23*, 73-85. doi:10.1080/0144341022000022942

Mather, N. & Woodcock, R. W. (2001). Examiner's manual. *Woodcock-Johnson III Tests of Achievement.* Itasca, IL: Riverside Publishing.

Mazzocco, M. M., Hanich, L. B., & Noeder, M. M. (2012). Primary school age students' spontaneous comments about math reveal emerging dispositions linked to later mathematics achievement. *Child Development Research*, *2012*, doi:10.1155/2012/170310.

McGraw, K. O., & Wong, S. P. (1996). Forming inferences about some intraclass correlation coefficients. *Psychological methods*, *1,* 30-46.

McGrew, K. S., Schrank, F. A., & Woodcock, R. W. (2007). Technical Manual. *Woodcock-Johnson III Normative Update.* Rolling Meadows, IL: Riverside Publishing.

McInerney, D. M., Cheng, R. W. Y., Mok, M. M. C., & Lam, A. K. H. (2012). Academic self-concept and learning strategies direction of effect on student academic achievement. *Journal of Advanced Academics*, *23*, 249-269. doi:10.1177/1932202X12451020

Metzger, S. R., Sonnenschein, S., & Galindo, C. (2018). *Elementary-age children's conceptions about math utility and their home-based math engagement.* Manuscript submitted for publication.

Mitchell, T. (1999). Changing student attitudes towards mathematics. *Primary Educator, 5*(4), 2-7.

Muis, K. R. (2004). Personal epistemology and mathematics: A critical review and synthesis of research. *Review of Educational Research, 74*, 317-377. doi:10.3012/00346543074003317

Murayama, K., Pekrun, R., Lichtenfeld, S., & vom Hofe, R. (2013). Predicting long-term growth in students' mathematics achievement: The unique contributions of motivation and cognitive strategies. *Child Development*, *84*, 1475-1490. doi:10.1111/cdev.12036

Musu-Gillette, L. E., Wigfield, A., Harring, J. R., & Eccles, J. S. (2015). Trajectories of change in students’ self-concepts of ability and values in math and college major choice. *Educational Research and Evaluation, 21*, 343-370. doi:10.1080/13803611.2015.1057161

Mutodi, P., & Ngirande, H. (2014). The influence of students’ perceptions on mathematics performance. A case of a selected high school in South Africa. *Mediterranean Journal of Social Sciences*, *5*, 431-445. doi:10.5901/mjss.2014.v5n3p431

Nagy, G., Watt, H. M., Eccles, J. S., Trautwein, U., Lüdtke, O., & Baumert, J. (2010). The development of students' mathematics self-concept in relation to gender: Different countries, different trajectories? *Journal of Research on Adolescence, 20*, 482-506. doi:10.1111/j.1532-7795.2010.00644.x

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences of Education. Washington, DC: National Academy Press.

National Mathematics Advisory Panel (2008). Foundations for success: The final report of the National Math Advisory Panel.

Odic, D. (2018). Children's intuitive sense of number develops independently of their perception of area, density, length, and time. *Developmental Science, 21*, e12533. doi:10.1111/desc.12533

Pajares, F., & Miller, M. D. (1994). Role of self-efficacy and self-concept beliefs in mathematical problem solving: A path analysis. *Journal of Educational Psychology, 86*, 193-203.

Perlmutter, J., Bloom, L., Rose, T., & Rogers, A. (1997). Who uses math? Primary children's perceptions of the uses of mathematics. *Journal of Research in Childhood Education, 12*, 58-70.

Pinxten, M., Fraine, B., Damme, J., & D'Haenens, E. (2010). Causal ordering of academic self-concept and achievement: Effects of type of achievement measure. *British Journal of Educational Psychology*, *80*, 689-709. doi:10.1348/000709910X493071

Preacher, K. J., Curran, P. J., & Bauer, D. J. (2006). Computational tools for probing interactions in multiple linear regression, multilevel modeling, and latent curve analysis. *Journal of Educational and Behavioral Statistics*, *31*, 437-448. doi: 10.3102/10769986031004437

Ramani, G. B & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development*, *79*, 375-394. doi:10.1111/j.1467-8624.2007.01131

Renninger, K., Hidi, S. E., & Krapp, A. E. (1992). *The role of interest in learning and development*. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Rozek, C. S., Hyde, J. S., Svoboda, R. C., Hulleman, C. S., & Harackiewicz, J. M. (2015). Gender differences in the effects of a utility-value intervention to help parents motivate adolescents in mathematics and science. *Journal of Educational Psychology, 107*, 195-206. doi:10.1037a0036981

Ryan, R. M., & Deci, E. L. (2000). Intrinsic and extrinsic motivations: Classic definitions and new directions. *Contemporary Educational Psychology*, *25*, 54-67. doi:10.1006/ceps.1999.1020

Schoenfeld, A. H. (1989). Explorations of students' mathematical beliefs and behavior. *Journal in Research in Mathematics Education, 20*, 338-355. doi:10.2307/749440

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics* (pp. 334-370). New York: Macmillan Publishing Company.

Seaton, M., Parker, P., Marsh, H. W., Craven, R. G., & Yeung, A. S. (2013). The reciprocal relations between self-concept, motivation and achievement: juxtaposing academic self-concept and achievement goal orientations for mathematics success. *Educational Psychology*, 24 pages. doi:10.1080/01443410.2013.825232.

Siegler, R. S. (2003). Implications of cognitive science research for mathematics education. In J. Kilpatrick, W. B. Martin, & D. E. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp.219-233). Reston, VA: National Council for Teachers of Mathematics.

Siegler, R. S. & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development*, *75*, 428-444. doi:10.1111/j.1467-8624.2004.00684.x

Simons, C. L., Metzger, S. R., & Sonnenschein, S. (2018). *Children's perceptions of key executive function processes: What they are and how children learn to use them.* Manuscript submitted for publication.

Simpkins, S. & Davis-Kean, P. E. (2005). The intersection between self-concepts and values: Links between beliefs and choices in high school. In J. E. Jacobs & S. D. Simpkins (Eds.), *Leaks in the pipeline to math, science, and technology careers* (Number 110, pp. 31-47). San Francisco, CA: Jossey-Bass.

Simpkins, S.D., Davis-Kean, P.E., & Eccles, J.S. (2006). Math and science motivation: A longitudinal examination of the links between choices and beliefs. *Developmental Psychology*, *42*, 70-83. doi:10.1037/0012-1649.42.1.70

Sobel, D. M., & Letourneau, S. M. (2015). Children's developing understanding of what and how they learn. *Journal of experimental child psychology*, *132*, 221-229. doi:10.1016/j.jecp.2015.01.004

Sonnenschein, S., Baker, L., & Garrett, A. (2004, December). Final Report of the 2004 Super Kids Camp. Sponsored by Parks and People Foundation.

Starkey, P., Spelke, E. S., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition*, *36*, 97-127.

Trautwein, U., Lüdtke, O., Köller, O., & Baumert, J. (2006). Self-esteem, academic self-concept, and achievement: How the learning environment moderates the dynamics of self-concept. *Journal of Personality and Social Psychology*, *90*, 334-349. doi:10.1037/0022-3514.90.2.334

U.S. Department of Education. (2018). *National Assessment of Education Progress.* National Center for Educational Statistics.

Wigfield, A., & Eccles, J. S. (2000). Expectancy–value theory of achievement motivation. *Contemporary Educational Psychology*, *25*, 68-81. doi:10.1006/ceps.1999.1015

Wigfield, A., Eccles, J. S., Fredricks, J. A., Simpkins, S., Roeser, R. W., & Schiefele, U. (2015). Development of achievement motivation and engagement. In R. M. Lerner (Ed.) & M. E. Lamb (Vol. Ed.), *Handbook of child psychology and developmental science: Vol. 3 socioemotional processes* (7th ed., pp. 1-44). Hoboken, NJ: John Wiley & Sons, Inc.

Wigfield, A., Eccles, J. S., Roeser, R. W., & Schiefele, U. (2008). Development of achievement motivation. In W. Damon, R. M. Lerner, D. Kuhn, R. S. Siegler, & N. Eisenberg (Eds.), *Child and adolescent development: An advanced course* (pp. 406-434). Hoboken, NJ: John Wiley & Sons, Inc.

Wigfield, A., Eccles, J. S., Schiefele, U., Roeser, R. W., & Davis-Kean, P. (2006). Development of achievement motivation. In W. Damon & R. M. Lerner (Series Eds.) & D. Kuhn & R. Siegler (Vol. Eds.), *Handbook of child psychology: Vol. 3 Social, Emotional, and Personality Development* (6th ed., pp. 933-1002). New York: Wiley.

Wigfield, A., Eccles, J. S., Yoon, K. S., Harold, R. D, Arbreten, A. J. A., Freedman-Doan, C. & Blumenfeld, P. C. (1997). Change in children's competence beliefs and subjective task values across the elementary school years: A 3-year study. *Journal of Educational Psychology*, *89*, 451-469. doi:10.1037/0022-0663.89.3.451

Wigfield, A., & Guthrie, J. T. (1997). Relations of children's motivation for reading to the amount and breadth or their reading. *Journal of Educational Psychology*, *89*, 420-432. doi:10.1037/0022-0663.89.3.420

Wigfield, A. & Meece, J. (1988). Math anxiety in elementary and secondary school students. *Journal of Educational Psychology*, *80*, 210-216.

Woodcock, R. W., McGrew, K. S., & Mather, N. (2001). *Woodcock-Johnson III Tests of Achievement.* Itasca, IL: Riverside Publishing.

Woodcock, R. W., McGrew, K. S., Schrank, F. A., & Mather, N. (2001, 2007). *Woodcock-Johnson III Normative Update.* Rolling Meadows, IL: Riverside Publishing.

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*, *74*, B1-B11. doi:10.1016/S0010-0277(99)00066-9

Table 1
Children's Math Achievement Scores

Subtest or Cluster	Raw Score *M* (*SD*)	Standard Score *M* (*SD*)
Calculation subtest	11.22 (4.59)	105.54 (16.36)
Math Fluency subtest	35.27 (18.12)	101.50 (14.65)
Applied Problems subtest	31.20 (6.23)	112.37 (13.66)
Quantitative Concepts subtest	28.61 (6.08)	110.39 (15.45)
Concepts subtest	17.53 (3.20)	N/A
Number Series subtest	11.08 (3.37)	N/A
Broad Math Cluster	N/A	110.51 (16.13)
Brief Math Cluster	N/A	111.74 (16.23)
Math Calculations Cluster	N/A	104.58 (16.08)
Math Reasoning Cluster	N/A	113.74 (15.60)

Table 2

Calculation and Reasoning Cluster and Subtest Comparisons

Comparison	Calculation Subtest or Cluster Standard Score *M* (*SD*)	Calculation Subtest or Cluster Standard Score *M* (*SD*)	Significance Test
Math Calculation and Math Reasoning clusters	104.58 (16.08)	113.74 (15.60)	$t(103) = 7.66$, $p < .001$
Calculation and Applied Problems subtests	105.54 (16.36)	112.47 (13.69)	$t(102) = 5.34$, $p < .001$
Calculation and Quantitative Concepts subtests	105.54 (16.36)	110.58 (15.40)	$t(102) = 3.51$, $p = .001$
Math Fluency and Applied Problems subtests	101.50 (14.65)	112.37 (13.66)	$t(103) = 8.78$, $p < .001$
Math Fluency and Quantitative Concepts subtests	101.50 (14.65)	110.39 (15.45)	$t(103) = 6.40$, $p < .001$

Note. One child did not receive a Calculation subtest standard score, which impacted the *N* and means for comparisons.

Table 3

Aspects of Children's Math Utility Conceptions Predicting Children's Math Achievement

	β	t*	Sig.
Broad Math			
Math Concepts	.111	1.11	p = .269
Math Applicability	.070	0.67	p = .506
Math Utility	-.222	-1.59	p = .116
Productive Disposition	.351	2.59	p = .011
Parents' Education	.208	1.99	p = .050
Household Income	.196	1.94	p = .056
Math Calculations			
Math Concepts	.142	1.42	p = .159
Math Applicability	-.034	-0.32	p = .749
Math Utility	-.190	-1.35	p = .180
Productive Disposition	.301	2.21	p = .029
Parents' Education	.246	2.34	p = .022
Household Income	.183	1.80	p = .075
Math Reasoning			
Math Concepts	.104	1.02	p = .309
Math Applicability	.153	1.43	p = .155
Math Utility	-.174	-1.22	p = .227
Productive Disposition	.319	2.30	p = .024
Parents' Education	.161	1.50	p = .137
Household Income	.149	1.44	p = .155

Note. **df* was 92 for all regressions.

Table 4

Grade Groups Moderating the Relation between Aspects of Children' Math Utility Conceptions and Children's Math Utility Conceptions

	b	*t*	*p*
Broad Math			
Math Concepts	-5.876	-0.75	.455
Math Applicability	9.783	2.82	.006
Math Utility	-26.554	-2.61	.011
Productive Disposition	9.981	1.20	.232
Grade group	-53.394	-2.40	.018
Math Concepts X grade group	5.994	1.33	.186
Math Applicability X grade group	-6.487	-2.85	.005
Math Utility X grade group	16.390	2.47	.016
Productive Disposition X grade group	-1.904	-0.37	.713
Parents' Education	2.408	1.42	161
Household Income	2.456	1.74	.085
		Adjusted R^2 = .25	
Math Calculations			
Math Concepts	-3.433	-0.44	.661
Math Applicability	6.972	2.01	.047
Math Utility	-19.656	-1.94	.056
Productive Disposition	5.479	0.66	.509
Grade group	-48.999	-2.12	.030
Math Concepts X grade group	4.884	1.09	.279
Math Applicability X grade group	-5.026	-2.21	.029
Math Utility X grade group	11.936	1.80	.075
Productive Disposition X grade group	0.378	0.07	.942
Parents' Education	3.089	1.82	.072
Household Income	2.207	1.57	.120
		Adjusted R^2 = .22	
Math Reasoning			
Math Concepts	-3.484	-0.43	.670
Math Applicability	7.184	1.99	.050
Math Utility	-24.000	-2.26	.026
Productive Disposition	9.279	1.08	.285
Grade group	-49.765	-2.15	.034
Math Concepts X grade group	4.151	0.89	.378
Math Applicability X grade group	-4.289	-1.81	.074
Math Utility X grade group	14.963	2.16	.033
Productive Disposition X grade group	-1.892	-0.35	.726
Parents' Education	2.055	1.16	.249
Household Income	1.704	1.16	.249
		Adjusted R^2 = .15	

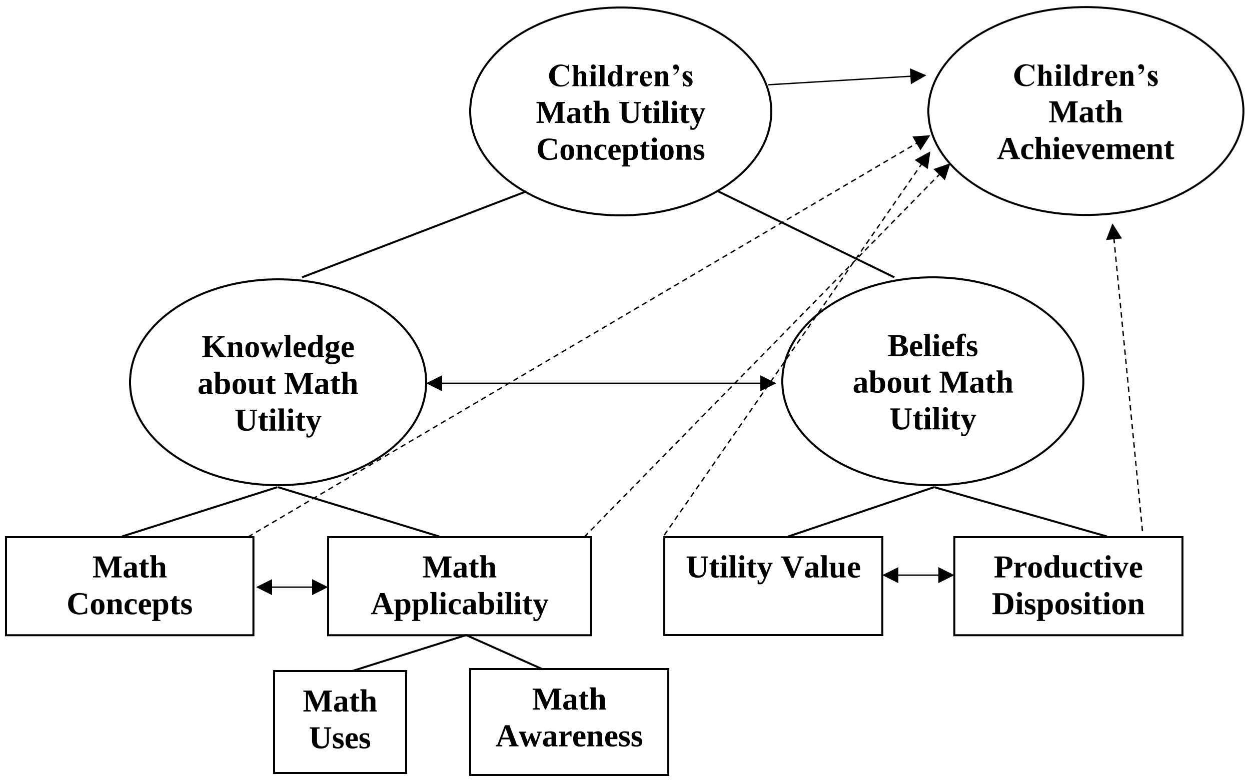

Figure 1. Hypothesized Relations Between Math Utility Conceptions and Math Achievement.

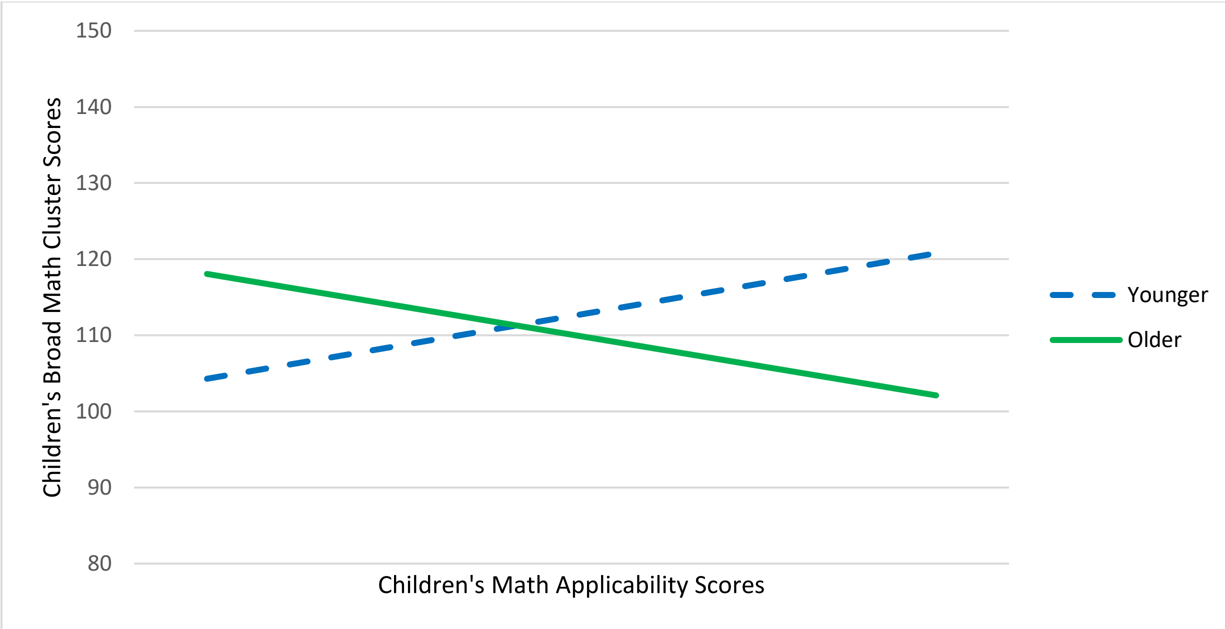

Figure 2. Children's Broad Math cluster scores as a function of children's math applicability scores for younger and older children.

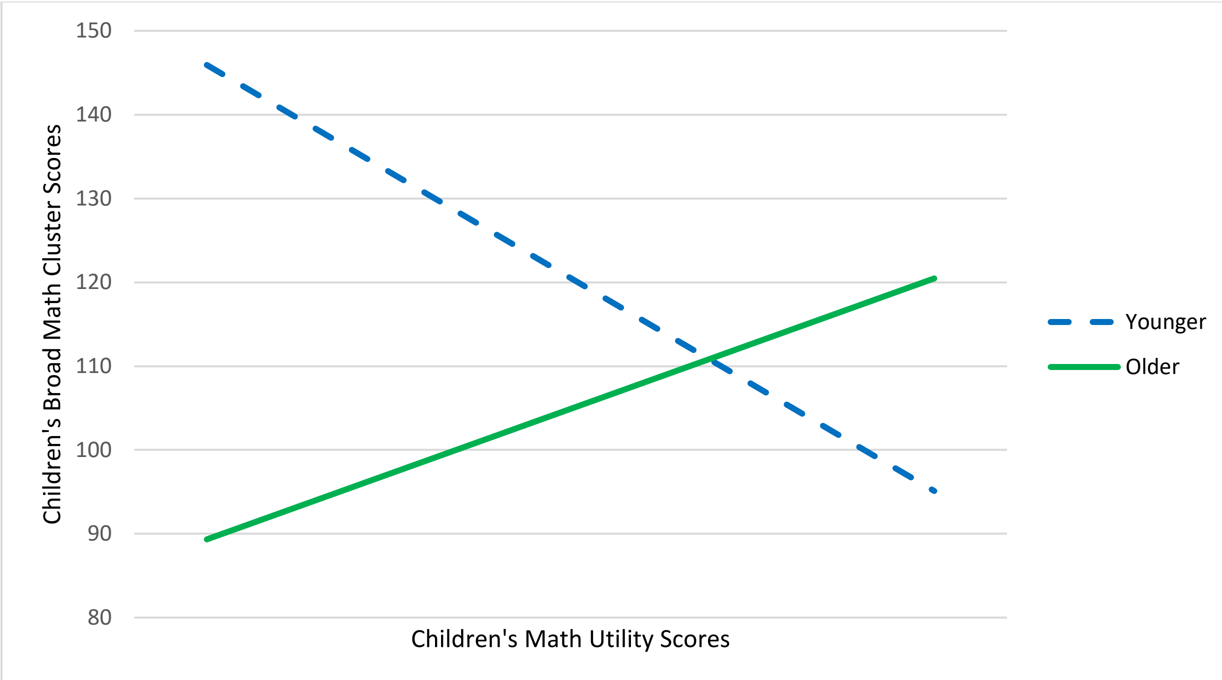

Figure 3. Children's Broad Math cluster scores as a function of children's math utility scores for younger and older children.

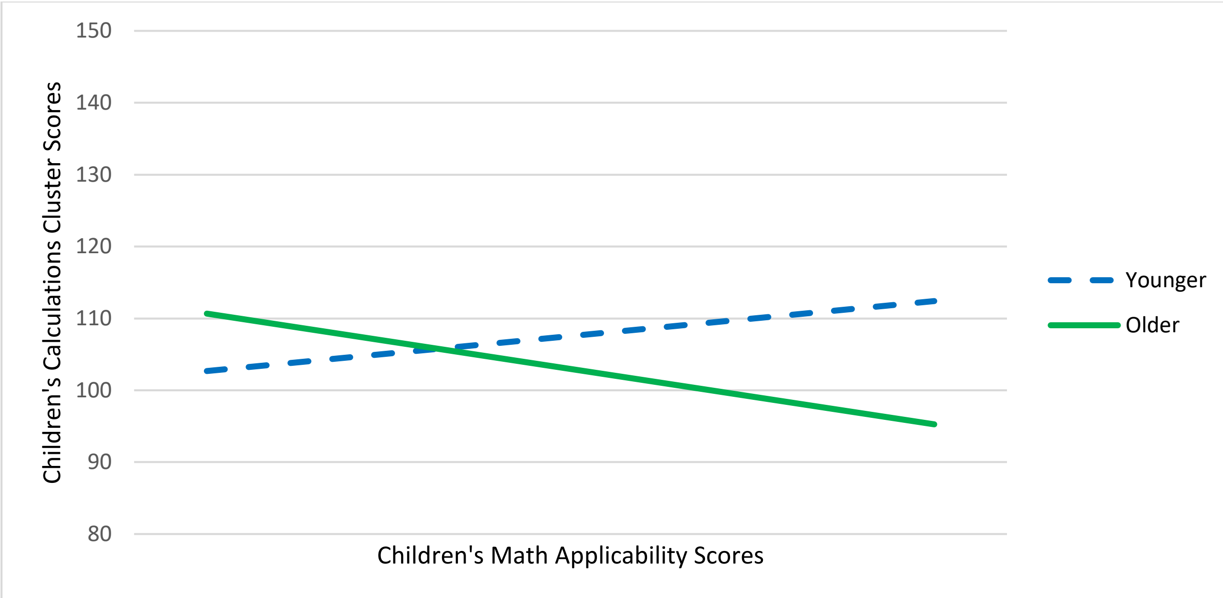

Figure 4. Children’s Math Calculations cluster scores as a function of children’s math applicability scores for younger and older children.

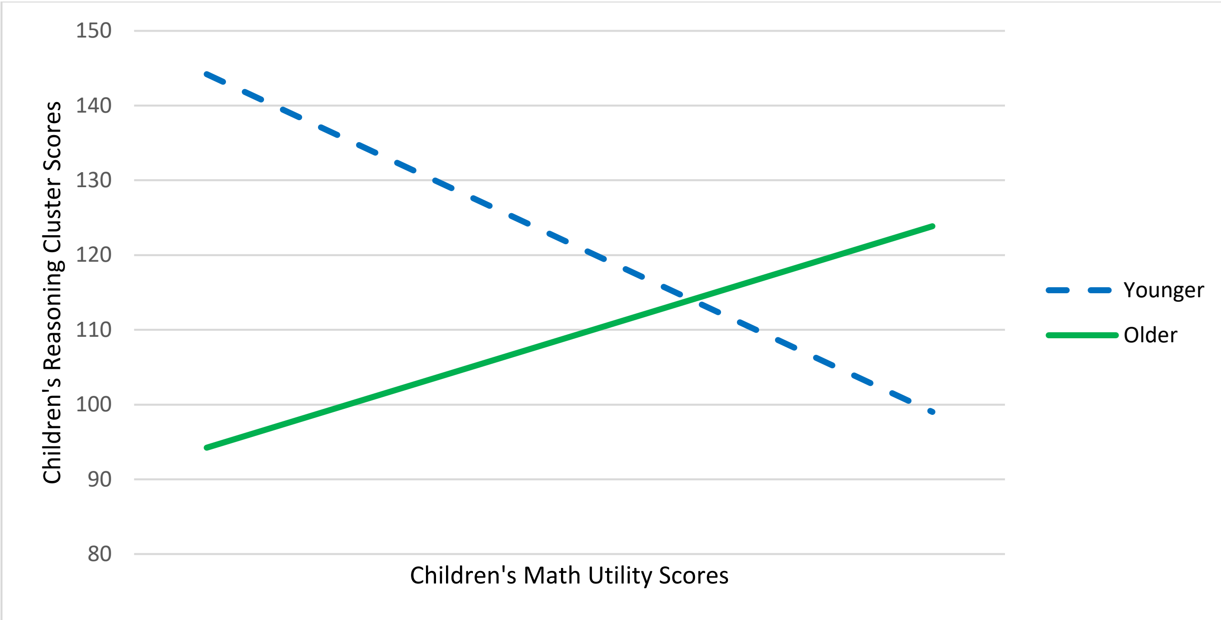

Figure 5. Children's Math Reasoning cluster scores as a function of children's math utility scores for younger and older children.

Chapter 5: General Conclusions

The primary goals of these three studies were to expand on current research in the way that math utility conceptions are measured and to gain a better understanding of children's math conceptions, how they are acquired, and how they impact achievement. Children were able to articulate knowledge and beliefs about math utility. There were many individual differences, but also some common themes in their responses. Overall, children view math primarily as something learned and used in school. However, aspects of their home environment can impact these conceptions.

Results from Paper 2 supported the notion that parents play an important role in the development of children's knowledge and beliefs about the usefulness of math. More specifically, parents' conceptions can have the most impact on children's conceptions for children who engage less frequently in math activities at home and more frequently see their parents using math. Understanding children's conceptions about how math is used and how the home environment may shape these views could guide future targeted interventions for elementary-age children to improve math utility knowledge and beliefs.

The third study provided evidence that children's knowledge and beliefs about the usefulness of math play a role in their math achievement, but this relation is complex. Results supported the notion that productive disposition towards math is associated with several areas of children's math achievement and that children's math utility conceptions relate to children's math reasoning skills. However, for math applicability and math utility value, the relations between math utility conceptions and achievement is different for younger and older elementary school-aged children.

Together these three studies add to the current literature on math utility conceptions in two overarching ways. First, these studies suggest that parents and the home environment may have a substantial impact on young children's math utility conceptions. Interventions with parents may be a very effective way to influence children's conceptions. Effective home-based interventions would likely be ones in which parents are given specific tools and examples of how to demonstrate to their children the ways that they use math in their daily lives.

Second, these studies suggest a link between children's knowledge and beliefs about math and their ability to use math to solve real-world problems. This association highlights the importance of considering children's math utility conceptions as a factor for improving children's math proficiency. More specifically, when developing curriculum for elementary-aged children, educators could more intentionally use real-world applications to teach math concepts and, in doing so, improve children's understanding of the importance of math in their daily lives. By increasing young children's knowledge of applications of math outside the school context and beliefs about the usefulness of math, parents and educators could help to increase children's math proficiency.

Appendix A
Children's Math Utility Conceptions Questionnaire

Interview Date: ________/_________/_________ Grade: ________________________

Interviewer: ______________________________ Gender: ______Boy ______Girl

Directions: I'm going to ask you some questions about math. There are no right or wrong answers. I only want to know what you think about math.

1. What is math? (*What do you think math is? How would you describe math to someone who had never heard of it?*)

____

____

____

____

2. a. Who uses math?

____

____

b. *For each of the people child lists, ask:* How does ______ use it?

Person:
1.____
2.____
3.____
4.____

3. How do you learn math? (*If child does not know or does not respond, probe by suggesting: teachers, parents, friends, things in class*)

____

____

____

____

Math Home Engagement

Directions: Now I'm going to ask you about things you may do at home. To answer some of these questions, you will use this number scale (*put scale on the table in front of the child*): never or almost never, "1 star," "2 stars," sometimes, "3 stars," "4 stars," and every day or almost every day, "5 stars." (*For each question, if the child responds "2" through "5", ask the follow up question. If the child responds "1," continue to the next question.*)

How often do you…?

	1 Never/ Almost Never	2	3 Sometimes	4	5 Every day/ Almost Every Day
1. Do math at home? What kind of math do you do at home? ____	1	2	3	4	5
2. Play board games at home? What board games do you play? (*Try to get them to list at least three*) ____	1	2	3	4	5
3. Play math games on the computer/iPad at home? What computer math games do you play? ____	1	2	3	4	5

	1 Never/ Almost Never	2	3 Sometimes	4	5 Every day/ Almost Every Day
4. Play games or sports where you keep score? What games or sports do you play where someone keeps score? ____________________________________ ____________________________________ ____________________________________ ____________________________________ ____	1	2	3	4	5
5. Play card games? What card games do you play? ____________________________________ ____________________________________ ____________________________________ ____________________________________ ____	1	2	3	4	5
6. Play video games, like on Playstation, Xbox, the Wii, or Nintendo DS at home? What video games do you play? ____________________________________ ____________________________________ ____________________________________ ____________________________________ ____	1	2	3	4	5
7. Help with cooking at home? How do you help with cooking? ____________________________________ ____________________________________ ____________________________________	1	2	3	4	5

_________________________________ ____					
8. Help at the grocery store? How do you help at the grocery store? _________________________________ _________________________________ _________________________________ _________________________________ ____	1	2	3	4	5
	1 Never/ Almost Never	2	3 Sometimes	4	5 Every day/ Almost Every Day
9. Watch math TV shows? Which math TV shows do you watch? _________________________________ _________________________________ _________________________________ _________________________________ ____	1	2	3	4	5
10. Use or play with money?	1	2	3	4	5
11. Use math workbooks or flashcards that are not part of your homework at home?	1	2	3	4	5
12. Play or build with blocks or Legos?	1	2	3	4	5
13. Use maps or globes at home?	1	2	3	4	5
14. Use a calculator at home?	1	2	3	4	5
15. Do puzzles at home?	1	2	3	4	5

Math Applicability

Directions: Now I want to know what you think about some different activities. Again, there are no right or wrong answers. I only want to know what you think.

For each activity, say the following:
Some children think math is used when they ________________, some think math is not used at all. Do you think math is used when you _________________? (*If child responds "Yes," ask:* "How is math used in that activity?")

A. Play board games, like ______ (*list one of the board games they mentioned above. If they did not list any, use "Candy Land or Chutes and Ladders."*) How is math used in (specify name of game)? ____ ____ ____ ______	Yes	No
B. Cook How is math used in cooking? ____ ____ ____ ______	Yes	No
C. Go to the grocery store How is math used when you go to the grocery store? ____ ____ ____ ______	Yes	No
D. Keep score in games or sports How is math used to keep score in games or sports? ____ ____ ____ ______	Yes	No
E. Play card games How is math used in playing cards?	Yes	No

____		

Some children think math is used when they __________________, some think math is not used at all. Do you think math is used when you ___________________? (*If child responds "Yes," ask:* "How is math used in that activity?")

F. Play with blocks or Legos How is math used when you with blocks or Legos? ____	Yes	No
G. Play video games How is math used in playing video games? ____	Yes	No
H. Use or play with money How is math used when using or playing with money? ____	Yes	No
I. Play with puzzles How is math used in playing with puzzles? ____	Yes	No
J. Use maps or a globe How is math used when using maps or a globe? ____	Yes	No

Beliefs about Math Utility

Directions: I am going to read you some statements saying how some students feel about math. To answer the next questions, you will use this number scale (*put scale on the table in front of the child*). Listen carefully and tell me if the person is not at all like you, "1 star," "2 stars," a little like you, "3 stars," "4 stars," or very much like you, "5 stars." There are no right or wrong answers. I only want to know how you feel about math. Let's do some examples.

Say, I (like ice cream). Is that not at all like you, "1 star," "2 stars," a little like you, "3 stars," "4 stars," or very much like you, "5 stars?"

Examples:	Not at all like me		A little like me		Very much like me
1. I like ice cream.	1	2	3	4	5
2. I like to swim.	1	2	3	4	5
3. I like spinach.	1	2	3	4	5

For each statement, say, (I need to learn math to do well in school). Is that not at all like you, "1 star," "2 stars," a little like you, "3 stars," "4 stars," or very much like you, "5 stars?"

	1 Not at all like me	2	3 A little like me	4	5 Very much like me
Subjective Task Value- Interest					
1. Math is interesting.	1	2	3	4	5
2. I like math.	1	2	3	4	5
3. I like to get math toys or math games as presents.	1	2	3	4	5
4. Math is fun.	1	2	3	4	5
5. I like to do math activities in my free time.	1	2	3	4	5
Subjective Task Value- Usefulness/Importance					
6. Math is useful to me for things other than school.	1	2	3	4	5
7. Math is useful to everyone.	1	2	3	4	5

8. I think people use math all the time in their lives.	1	2	3	4	5
9. Math is more useful than other subjects.	1	2	3	4	5
10. I need to learn math to do well in school.	1	2	3	4	5
11. It is important for me to learn math.	1	2	3	4	5
12. My parents think it is important for me to learn math.	1	2	3	4	5
13. I think it is important for everyone to learn math.	1	2	3	4	5
14. It is important for me to do well in math.	1	2	3	4	5
<u>Productive Disposition</u>					
15. Math is a subject I can use in my life.	1	2	3	4	5
16. Learning math is worth my time.	1	2	3	4	5
17. Working hard in math is worth my time.	1	2	3	4	5
18. Working hard in math helps me do better in math.	1	2	3	4	5
19. I am good at math.	1	2	3	4	5
20. I am better at math than my classmates.	1	2	3	4	5
21. I am better at math than my other subjects.	1	2	3	4	5
22. I know that I will do well in math this year.	1	2	3	4	5
23. Math is easy for me.	1	2	3	4	5
24. When doing math is difficult for me, working harder helps me solve it.	1	2	3	4	5

Appendix B

Parents' Math Utility Conceptions Questionnaire

Parents' Math Conceptions Questionnaire

Please answer the following questions about yourself. Your answers will be kept strictly confidential and your name will not be associated with your final responses.

Name:____

Relation to Child: ____ Mother ____ Father ____Other, please specify:____________________

Date of Birth: ______/______/_______

Gender ______Male ______Female

Race/ethnicity (Select all that apply):

_____African American/Black

_____European American/ White

_____Hispanic/Latino:
From where: _________________________

_____Asian/Pacific Islander:
From where:_________________________

_____Other, *please specify*:____

Date questionnaire completed: ______/______/_______

Please answer the following questions as thoroughly as you can.

1. What is math? (What do ***you*** think math is; how would you explain what math is to someone else?)

2. What is the best way to help your child learn math?

3. What do you believe is the role of parents/home in helping children learn math?

4. What do you believe is the role of teachers/school in helping children learn math?

For the following, please circle the best answer.

5. How often does your child do ***any*** math activities at home?

1	2	3	4	5
Never/ almost never	Once or twice a month	Once a week	A few times a week	Everyday or almost everyday

On days when your child does math activities at home, for about how many **minutes per day** does your child spend doing math activities at home? ______________________minutes per day

6. How often does your child do the following activities at home **(Circle one answer per row)**?

		1 Never/ almost never	**2 Once or twice a month**	**3 Once a week**	**4 Several times a week**	**5 Everyday or almost everyday**
a.	Play board games	1	2	3	4	5
b.	Play math games on the computer/iPad	1	2	3	4	5
c.	Play games or sports where someone keeps score	1	2	3	4	5
d.	Play card games	1	2	3	4	5
e.	Play video games (Playstation, Xbox, Nintendo, etc.)	1	2	3	4	5
f.	Help with cooking	1	2	3	4	5
g.	Help at the grocery store	1	2	3	4	5
h.	Watch math TV programs	1	2	3	4	5
i.	Use or play with money	1	2	3	4	5
j.	Use math workbooks/ flashcards (***not*** assigned by teacher)	1	2	3	4	5
k.	Play or build with blocks or Legos	1	2	3	4	5
l.	Use maps or globes	1	2	3	4	5
m.	Use a calculator	1	2	3	4	5
n.	Play with jigsaw puzzles	1	2	3	4	5

7. On a typical weekday, approximately how many minutes does your child spend on math homework? __________minutes

8. On a typical weekday, approximately how many minutes do you help your child with math homework? __________minutes

For the following, please circle the best answer.

9. How important is it that your child does math activities at home?

1	2	3	4	5
Not very		Somewhat		Very

10. How important is it that you help your child with math?

1	2	3	4	5
Not very		Somewhat		Very

11. How much do ***you*** enjoy math?

1	2	3	4	5
Not at all		Somewhat		Very Much

12. How often does your child see **you** doing math?

1	2	3	4	5
Never/ almost never	Once or twice a month	Once a week	Several times a week	Everyday or almost everyday

For the following, please circle the extent to which you agree or disagree with the following statements.

	1 Strongly Disagree	**2 Disagree**	**3 Neutral**	**4 Agree**	**5 Strongly Agree**
13. Math is interesting.	1	2	3	4	5
14. I like math.	1	2	3	4	5
15. I like to get math games as presents.	1	2	3	4	5
16. Math is fun.	1	2	3	4	5
17. I like to do math activities in my free time.	1	2	3	4	5
18. Math is useful to me.	1	2	3	4	5
19. Math is useful to everyone.	1	2	3	4	5
20. I think people use math all the time in their lives.	1	2	3	4	5
21. Math is more useful than other subjects.	1	2	3	4	5
22. Learning math is important for doing well in school.	1	2	3	4	5

	1 Strongly Disagree	2 Disagree	3 Neutral	4 Agree	5 Strongly Agree
23. It was important for me to learn math in school.	1	2	3	4	5
24. My parents thought it was important for me to learn math.	1	2	3	4	5
25. I think it is important for everyone to learn math.	1	2	3	4	5
26. It was important for me to do well in math in school.	1	2	3	4	5
27. Math is a subject I can use in my life.	1	2	3	4	5
28. Learning math was worth my time.	1	2	3	4	5
29. Working hard in math was worth my time.	1	2	3	4	5
30. Working hard in math helped me do better in math.	1	2	3	4	5
31. I am good at math.	1	2	3	4	5
32. I am better at math than my peers.	1	2	3	4	5
33. I was better at math than other academic subjects.	1	2	3	4	5
34. Math is easy for me.	1	2	3	4	5
35. When doing math is difficult for me, working harder helps me solve it.	1	2	3	4	5

Finally, we'd like to know about how you believe math can be used (or not) in different contexts. For the following, please circle "Yes" or "No." If you respond "Yes" to any question, please describe how math is used with as much detail as possible.

36. Can math be used when decorating a home?	**Yes**	**No**
If "yes", how is math used? ____________________ ____________________ ____________________		

37. Can math be used in the kitchen?	**Yes**	**No**
If "yes", how is math used? ____________________ ____________________ ____________________		

38. Can math be used when gardening/mowing the lawn?	**Yes**	**No**
If "yes", how is math used? ______________________________ ______________________________ ______________________________		
39. Can math be used when using a cell phone?	**Yes**	**No**
If "yes", how is math used? ______________________________ ______________________________ ______________________________		
40. Can math be used planning a party or get-together?	**Yes**	**No**
If "yes", how is math used? ______________________________ ______________________________ ______________________________		
41. Can math be used when making art?	**Yes**	**No**
If "yes", how is math used? ______________________________ ______________________________ ______________________________		
42. Can math be used when travelling?	**Yes**	**No**
If "yes", how is math used? ______________________________ ______________________________ ______________________________		
43. Can math be used when playing or watching sports?	**Yes**	**No**
If "yes", how is math used? ______________________________ ______________________________ ______________________________		
44. Can math be used when making/listening to music?	**Yes**	**No**
If "yes", how is math used? ______________________________ ______________________________ ______________________________		
45. Can math be used at a restaurant?	**Yes**	**No**
If "yes", how is math used? ______________________________ ______________________________ ______________________________		

Appendix C
Assumptions of Normality and Correlation Matrix
Chapter 3

Assumptions for the linear regression, linearity between independent and dependent variables, homoscedasticity of the errors, independence of the errors, and normality of the error distribution, predicting overall children's overall math utility conceptions were tested, using the method described by Nau (2005). There was no evidence of a violation the first three assumptions. Some of the math utility conceptions variables were significantly non-normally distributed, but because the linearity between independent and dependent variables assumption was not violated, it is still appropriate to use these variables in regressions.

Correlations Between Chapter 3 Variables

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Children's Overall Math Utility Conceptions (1)	1	$.27^{**}$	$.43^{**}$	$.56^{**}$	$.75^{**}$	$.61^{**}$	.15	$.30^{**}$	$.33^{**}$
Parents' Overall Math Utility Conceptions (2)		1	-.01	.17	.19	$.25^{*}$	$.20^{*}$	.14	.19
Children's Math Concepts Score (3)			1	$.34^{**}$	.01	-.09	< .01	$.28^{**}$	$.34^{**}$
Children's Math Applicability Score (4)				1	.17	-.05	-.01	$.42^{**}$	$.38^{**}$
Children's Math Utility Score (5)					1	$.70^{**}$	.14	.08	.13
Children's Productive Disposition Score (6)						1	$.24^{*}$	< -.01	.04
Mean Frequency of Home Math Engagement (7)							1	.04	$.21^{*}$
Child Grade Group (8)								1	$.21^{*}$
Overall Length of Utterance (9)									1

Note. *$p < .05$, ** $p < .01$

Nau, R. F. (2005). *Testing the assumptions of linear regression* [Web Document]. Retrieved from Lecture Notes Online Web Site: http://people.duke.edu/~rnau/411home.htm

Chapter 4

Assumptions for the linear regression, linearity between independent and dependent variables, homoscedasticity of the errors, independence of the errors, and normality of the error distribution, predicting overall children's math achievement were tested, using the method described by Nau (2005). There was no evidence of a violation of the first three assumptions. Some of the math utility conceptions variables were significantly non-normally distributed, but because the linearity between independent and dependent variables assumption was not violated, it is still appropriate to use these variables in regressions.

Correlations Between Chapter 4 Variables

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Children's Overall Math Utility Conceptions (1)	1	.43**	.56**	.75**	.61**	.09	.06	.13	.30**
Children's Math Concepts Score (2)		1	.34**	.01	-.09	.11	.13	.13	.28**
Children's Math Applicability Score (3)			1	.17	-.05	.06	-.02	.16	.42**
Children's Math Utility Score (4)				1	.70**	-.01	-.02	.02	.08
Children's Productive Disposition (5)					1	.17	.14	.16	< -.01
Broad Math (6)						1	.91**	.90**	-.13
Math Calculations (7)							1	.70**	-.20*
Math Reasoning (8)								1	-.02
Child Grade Group (9)									1

Note. **p* < .05, ** *p* < .01

Nau, R. F. (2005). *Testing the assumptions of linear regression* [Web Document]. Retrieved from Lecture Notes Online Web Site: http://people.duke.edu/~rnau/411home.htm